Mathematics 6

Problem Solving Activities

EYMOUR PUBLICATIONS

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Problem Solving Activities

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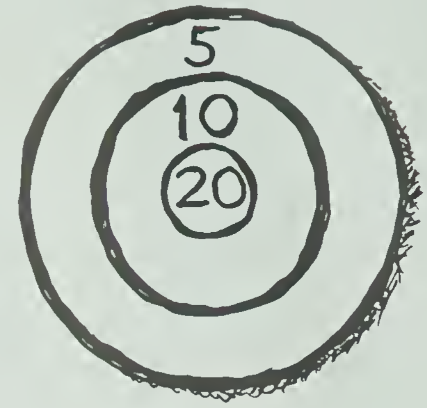
UNIT 1 Organizing Information

Problem:

Rick takes 3 shots at the target and hits it every time. How many different total scores are possible?



Solution:

Make an orderly list of all possible scores.

Scores For Each of Three Shots			Total Score
20	20	20	60
20	20	10	50
20	20	5	45
20	10	10	40
20	10	5	35
20	5	5	30
10	10	10	30
10	10	5	25
10	5	5	20
5	5	5	15

There are 9 different scores possible.

Make a list to help you solve the problem.

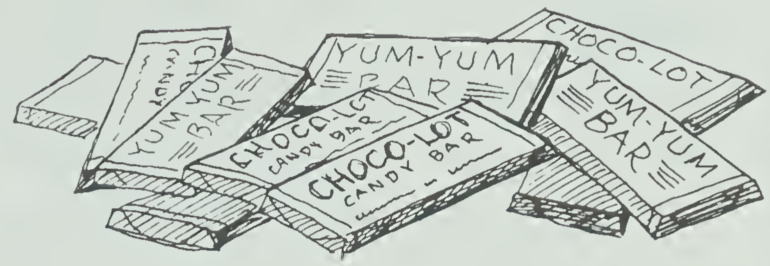
1. Today, Freda's mother is 3 times as old as Freda. Freda was born when her mother was 22. How old is Freda today?

Freda's Age	0	1	2			
Mother's Age	22	23	24			

2. In a one-child family, there are two possibilities. Either the child is a boy or it is a girl. In a two-child family, there are three possibilities. The children could be 2 boys, 1 boy and 1 girl, or 2 girls. List all the possibilities for a family of 4 children.

3. Jeff needs exactly one dollar's worth of postage stamps to mail a package. Jeff has lots of 10¢ stamps and 15¢ stamps but no other stamps. Show 4 different combinations of stamps that Jeff could put on the package.

4. Darlene bought some 30¢ and some 60¢ candy bars. Altogether she bought 10 candy bars and paid a total of $4.20. How many candy bars at each price did she buy?



5. With one digit, you can make only one whole number. With two digits, such as 1 and 2, you can make two 2-digit numbers (12 or 21). With the digits 1, 2, and 3 you could make six 3-digit whole numbers. What are all of the different 4-digit whole numbers you can make using the digits 1, 2, 3, and 4? Do not use the same digit twice in any of your numbers.

6. Jill's bug collection contains 12 beetles and spiders. Beetles are worth 10¢ and spiders are worth 20¢. Altogether Jill's collection is worth $1.70. How many beetles and how many spiders does Jill have?

7. Limericks contain 5 lines and sonnets contain 14 lines. Linda memorized sonnets and limericks with a total of 112 lines. How many sonnets and how many limericks did Linda memorize?

8. Janice has a 5-card hand containing only 7s, 8s, and 9s. The sum of the numbers in her hand is 41. How many 7s, 8s, and 9s does Janice have?

9. Duke and Donna each have 10 coins. All coins are either dimes or quarters. Duke has 5 quarters. Donna has 60¢ more than Duke. How many quarters does Donna have?

Problem:

At the First-Rate Bank, there is a service charge of $1.00 per month and an extra charge of 25¢ for each check written. At the Top-Terms Bank, there is a service charge of $2.00 per month and an extra charge of 10¢ for each check written. How many checks would you have to write so that the rates at the Top-Terms Bank would be better?

Solution:

Make a table to compare the rates for writing any number of checks.

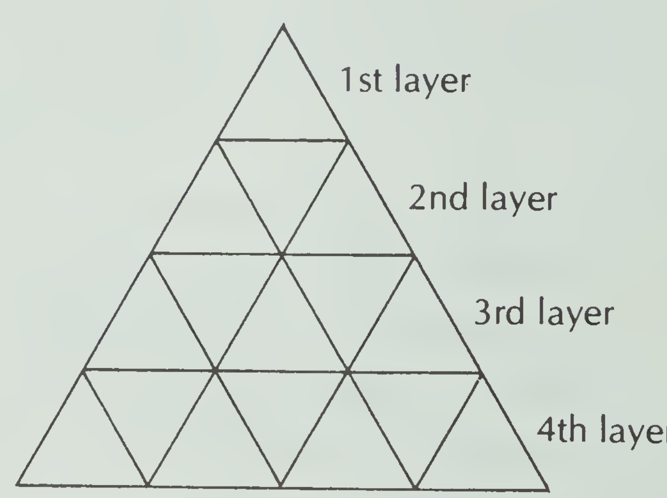
Number of checks	0	1	2	3	4	5	6	7
First-Rate Bank	$1.00	$1.25	$1.50	$1.75	$2.00	$2.25	$2.50	$2.75
Top-Terms Bank	$2.00	$2.10	$2.20	$2.30	$2.40	$2.50	$2.60	$2.70

The rates are better at Top-Terms Bank when 7 or more checks are written.

Make a table to help you solve each problem.

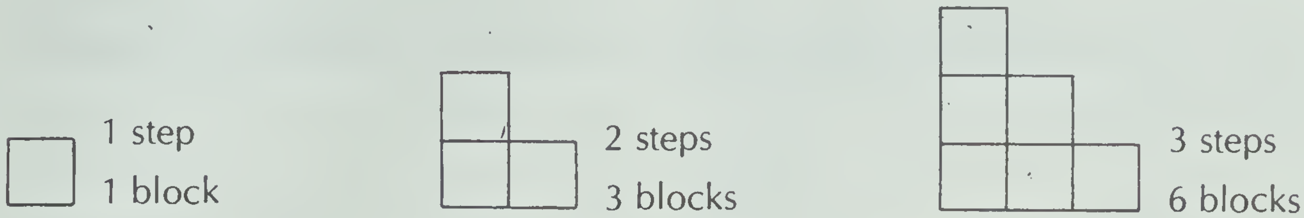
1. Nick has $40.00 and spends $1.50 each day. Bick has only $9.00 but he saves $1.50 each day. In how many days will Bick have more money than Nick?

2. In the large triangle, there is 1 small triangle in the first layer, 3 in the second layer, and 5 small triangles in the third layer. How many small triangles would there be in the tenth layer?



3. Glenna has 2 parents and 4 grandparents. Each of Glenna's grandparents had 2 parents. So, if they were living, Glenna would have 8 great grandparents. If Glenna thinks back *more* generations, how many great, great, great, great grandparents would Glenna have?

4. Suppose we wish to make a stairway using blocks. How many blocks would be needed to make a 10-step stairway?



5. Suppose that tickets for a rock concert cost 5¢ for the first ticket, 10¢ for the second ticket, 15¢ for the third ticket, and so on. If you bought 10 tickets, how much would you pay altogether?

6. One cell divides into 2 cells. Each of those 2 cells divides into 2 more cells for a total of 4 cells. Those 4 cells each divide into 2 more cells. How many cells will there be after the tenth division of cells?

7. A ball is dropped from the CN Tower in Toronto. It falls 5 m in the first second. It speeds up so that it falls 15 m in the *second* second for a total fall of 20 m. It continues to speed up and falls a distance of 10 m more for each second of time than it fell in the previous second. The ball is dropped from a place 500 m high on the tower. How long would it take to reach the ground?

Second of Fall	1st Second	2nd Second	3rd Second			
Distance fallen in that second (meters)	5	15	25			
Total Distance fallen (meters)	5	20	45			

8. Rae starts on a bicycle trip at 8:00 A.M. at the rate of 12 km/h. Two hours later, Myra starts out at the rate of 18 km/h. If both girls maintain their speeds, at what time will Myra catch up to Rae?

UNIT 2 Guessing and Testing

A problem can often be solved by guessing an answer and testing to see if the guess is a solution.

Problem:
Sue can't resist donuts. She saw that plain donuts were on sale for 35¢ each and fancy donuts for 50¢ each. Sue bought a dozen donuts and paid \$4.65. How many fancy donuts did she buy?

Solution:
A table helps to record and check the guesses in an orderly manner.

	1st Guess:	2nd Guess:	3rd Guess:
Plain Donuts	$6 \times \$0.35 = \$2.10$	$8 \times \$0.35 = \$2.80$	$9 \times \$0.35 = \$3.15$
Fancy Donuts	$6 \times \$0.50 = \$3.00$	$4 \times \$0.50 = \$2.00$	$3 \times \$0.50 = \$1.50$
Total	12 \$5.10	12 \$4.80	12 \$4.65
	Too much!	Closer!	Just right!

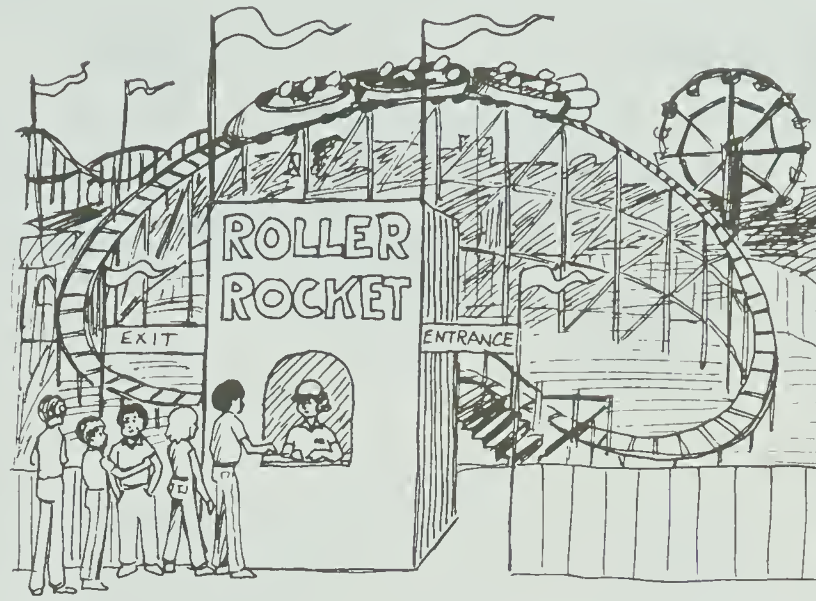
Sue bought 3 fancy donuts.

1. Pancho bought some cans of soup at 39¢ each and a jar of jam at \$1.95. Altogether he spent \$5.07. How many cans of soup did he buy?

2. Sesame snaps cost 30¢ each, and packs of gum cost 25¢. Brenda bought twice as many sesame snaps as packs of gum. Altogether she spent \$4.25. How many sesame snaps did she buy?

3. At the Glamour Boutique, a bottle of Allura Numero 75 perfume costs as much as two bottles of Exotic Mist. Mrs. Hampton bought one bottle of each and paid \$24.00. How much does a bottle of Allura cost?

4. At the Fun-For-All Amusement Park, Gerri went on 8 rides. Some of her rides were on the Roller-Rocket, and the others were on the Terrible Twist. Rides on the Roller-Rocket cost $0.90 and rides on the Terrible Twist cost $0.60 each. Altogether Gerri spent $6.30 on both kinds of rides. How many times did she go on the Roller-Rocket?



5. Apples cost 10¢ each, bananas cost 20¢ each, and peaches cost 30¢ each. Zeke bought eight pieces of fruit and spent $1.30. How many apples, bananas, and peaches did Zeke buy?

6. Miss Piggy ate 400 more kilograms of hogfood than Mr. Porky. Together they ate a total of 1400 kg of hogfood. Hogfood costs $2.00/kg. How much did it cost to feed Miss Piggy?

7. Marta has exactly 100 coins, all nickels and dimes. She counted her money and it came to $8.10. How many of each coin does Marta have?

8. Adult tickets for a movie cost $4.50 and children's tickets cost $2.00. There are 120 people at the movie. Ticket sales amounted to $440.00. How many children attended?

9. At the post office, Slim bought 30¢, 40¢, and 60¢ stamps. He bought the same number of 40¢ stamps as 60¢ stamps. He bought 15 stamps altogether for a total of $6.90. How many 30¢ stamps did he buy?

10. At Big Burger Barn, 6 children ordered milkshakes and hamburgers. Some had Big Burgers; some had Giant Burgers. There were 9 burgers ordered altogether. Every child had 1 milkshake. The total bill, including milkshakes and burgers, was $16.80. How many Giant Burgers were ordered?

BIG BURGER BARN	
Big Burger	$1.20
Giant Burger	$1.50
Milkshakes	$0.90

Problem:

Cora counts calories carefully. One day, when she couldn't resist, she ate 12 candies and cookies. Candies have 300 calories each; cookies have 250 calories each. Cora ate candies and cookies containing 3500 calories. How many cookies did Cora eat?

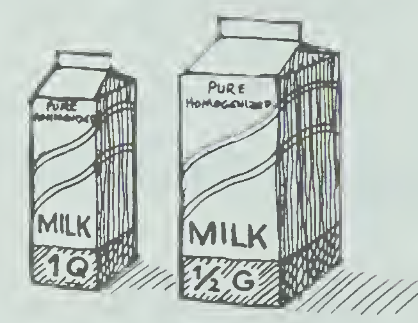
Solution:

	1st Guess:	2nd Guess:
Candies	$8 \times 300 = 2400$ calories	$10 \times 300 = 3000$ calories
Cookies	$4 \times 250 = 1000$ calories	$2 \times 250 = 500$ calories
Total	12 3400 calories	12 3500 calories

Too few! **Just right!**

Cora ate 2 cookies.

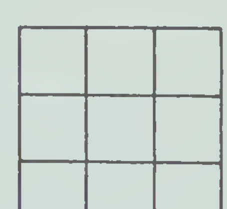
1. Mr. Nardi of Dairy Farm Market checked his supply of milk. He counted 120 containers of milk. Some of them were 1 quart containers; the rest were half-gallon containers. Altogether, there was a total of 190 quarts of milk in the two kinds of containers. How many were half-gallon containers?



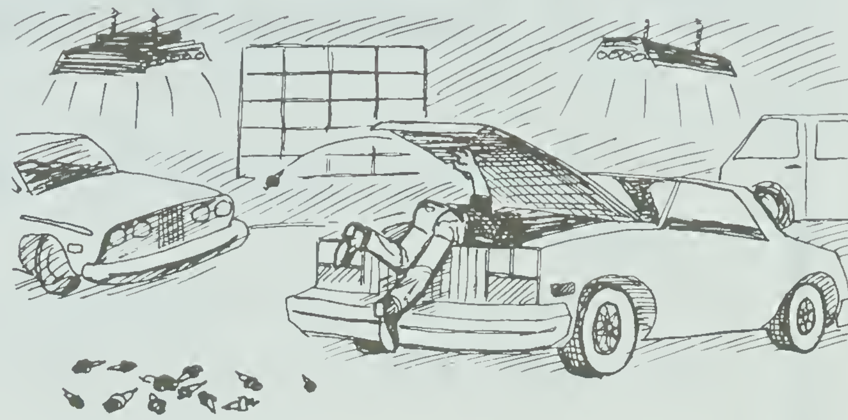
2. Two hundred thirty-five children go to Camp Fir Tree. There are 17 more girls than boys. How many are girls?

3. Joe got 10 more votes than Rita in the school election. Meg got twice as many votes as Joe. Altogether 282 votes were cast. How many votes did the winner get?

4. Arrange the digits 1 to 9 in a square grid so that every column, row, and diagonal has the same sum.



5. Jack's brother, Zack, does engine tune-ups. In one week, he replaced the spark plugs on 22 cars. Each car had either 4 or 6 spark plugs. Zack replaced a total of 116 spark plugs. How many of the cars had 6 spark plugs each?



6. An example of five consecutive numbers is 51, 52, 53, 54, and 55. The sum of five different consecutive numbers is 720. What is the largest number of these five?

7. Vic is half as old as I am. Mick is twice as old as I am. The sum of all of our ages is 49. How old am I?

8. Fly-By-Nite Airways has a fleet of 32 planes. The planes have either 2 engines or 4 engines. Altogether there is a total of 96 engines on all of the planes. How many of the planes have 4 engines?

9. A whole number has two digits. The second digit is two more than the first digit. The sum of the digits is 14. What is the number?

10. The sum of two whole numbers is 100. The difference between these two numbers is 32. What are the two whole numbers?

11. What three different whole numbers give the same result when they are added as when they are multiplied?

12. What two-digit number is equal to three times the product of its digits?

UNIT 3 Looking For Patterns

Once an orderly list of numbers is made, it is easier to look for a pattern.

Problem:
Multiply the number 4 by itself again and again.
What happens to the ones-place digit in the product?

Solution:
List all the products using 4s. Look for the pattern.

$$4 = \mathbf{4}$$
$$4 \times 4 = 1\mathbf{6}$$
$$4 \times 4 \times 4 = 6\mathbf{4}$$
$$4 \times 4 \times 4 \times 4 = 25\mathbf{6}$$
$$4 \times 4 \times 4 \times 4 \times 4 = 102\mathbf{4}$$

For an *even* number of 4s, the product has a **6** in the ones place. For an *odd* number of 4s, the product has a **4** in the ones place.

Find a pattern.

1. Double the number 3. Now double that number. Keep doubling until you find a pattern for the digits in the ones place. What is the pattern?

2. What pattern results in the ones-place digits of the products if we multiply the number 6 by itself over and over again?

3. Find patterns for the ones-place digits of the products when we multiply each of the following numbers by themselves over and over again:
   a. 3
   b. 7
   c. 9

4. Add 6 to itself over and over again. What is the pattern in the ones-place digits of the sums?

5. Study the patterns in the list of products below. What are the missing products?

$11 \times 99 = 1089$	$55 \times 99 =$
$22 \times 99 = 2178$	$66 \times 99 =$
$33 \times 99 = 3267$	$77 \times 99 =$
$44 \times 99 = 4356$	$88 \times 99 =$
	$99 \times 99 =$

6. Choose any 3-digit number and multiply it by 1001. Pick another 3-digit number and multiply by 1001. Look for number patterns. Predict the products if you multiply 345 and 627 by 1001.

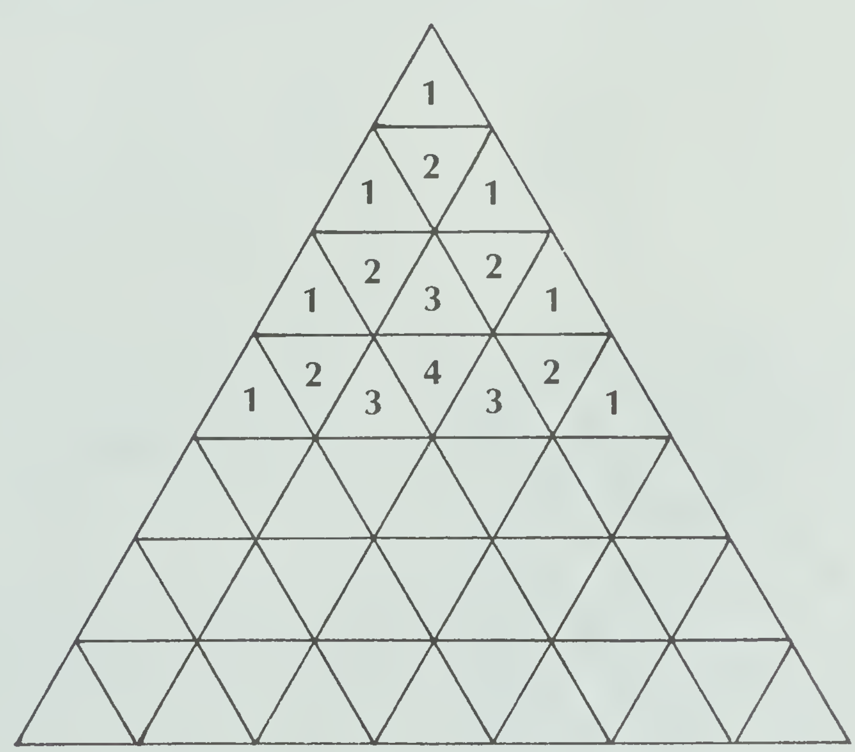
7. Study the dot patterns. How many dots would there be in all if there were 10 rows of dots?

Look at the triangular array at the right.

8. What is the middle number in the 9th row? 20th row?

9. How many numbers will be in the 9th row? 20th row?

10. What is the sum of the numbers in the 9th row? 20th row?



Problem:
At the Double-Dip Ice Cream Palace there are 5 flavors of ice cream. How many different double dips are possible? (Count two dips of the same flavor as one of the different double dips.)

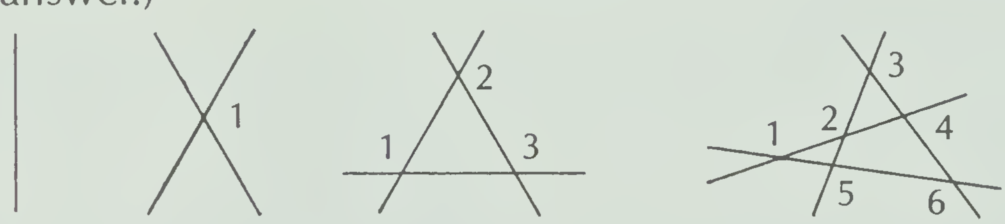
Solution:
Make a table and then look for a continuing pattern to solve the problem.

Number of			
Flavors	Double Dips		
1	1		*AA*
		+2	
2	3		*AA BB AB*
		+3	
3	6		*AA BB CC AB AC BC*
		+4	
4	10		*AA BB CC DD AB AC AD BC BD CD*
		+5	
5	15		*AA BB CC DD EE AB AC AD AE BC BD BE CD CE DE*

Fifteen double dips are possible.

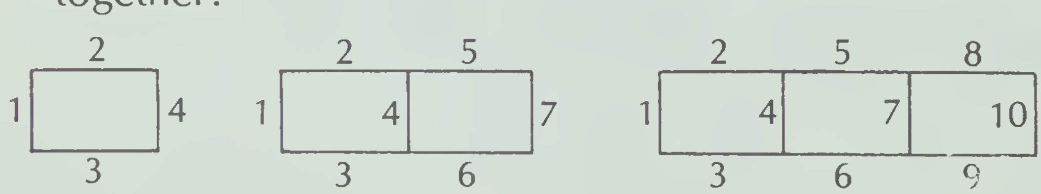
Use the pattern to solve the problem.

1. The greatest number of points at which 2 lines can meet is 1. The greatest number of points at which 3 lines can meet is 3. What is the greatest number of points at which 6 lines could meet? (Illustrate your answer.)



Lines	Points
1	0
2	1
3	3

2. One rectangle has 4 sides. If you arrange 2 rectangles together, you will see 7 sides. Join a third rectangle to the first two, and you will see 10 sides. How many sides would you see if you joined 12 rectangles together?



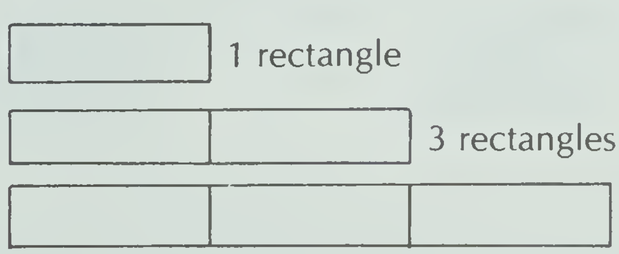
Rectangles	Sides Seen
1	4
2	7
3	10

3. With the digits 1 and 2, you can form exactly two different 2-digit numbers with no repetitions: 12 and 21. With the digits 1, 2, and 3, you can form exactly six different 2-digit numbers: 12, 13, 23, 21, 31, and 32. How many different 2-digit numbers can you form with the digits 1, 2, 3, and 4? How many different 2-digit numbers can be formed with the digits 1, 2, 3, 4, 5, and 6?

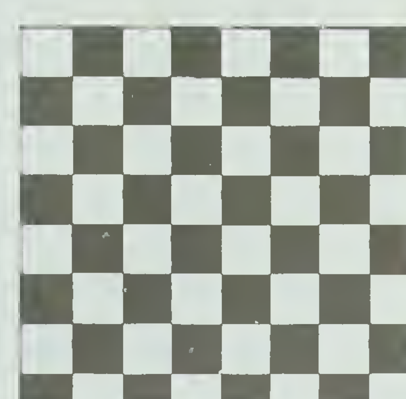
Numbers of digits	1	2	3	4
Different numbers	1	2	6	

4. One straight cut will divide a pie into 2 pieces. The greatest number of pieces from 2 straight cuts is 4. The greatest number of pieces from 3 straight cuts is 7. What is the greatest number of pieces that will result from 6 straight cuts?

5. When 2 rectangles are joined together, you can see 1 large rectangle and 2 smaller ones. When 3 rectangles are joined together, you can see 1 large rectangle, 2 smaller ones (each made up of 2 little rectangles), and 3 little rectangles. How many different-sized rectangles would there be if you arranged 8 rectangles in one long row?



6. On an 8 by 8 checkerboard, the largest rectangle which you have is the whole board, 8 by 8. The smallest rectangle which you could have is 1 by 1. How many different-sized rectangles are there on a checkerboard? (Assume a 5 by 3 rectangle to be the same as a 3 by 5 rectangle.)



UNIT 4 Too Many and Too Few Facts

Often there are more facts given in a problem than are needed to solve it.

Problem:
Tickets for the circus cost $6.00 for adults and $3.50 for children. On opening day 6000 people attended the circus. Of these people, 4500 were children. Mr. and Mrs. Stone went to the circus on opening day with their 3 children. How much did the tickets cost Mr. Stone?

Solution:
Use only the facts needed to solve the problem.

Cost for 2 adults:	$2 \times \$6.00 =$	$12.00
Cost for 3 children:	$3 \times \$3.50 =$	$10.50
Total cost:		$22.50

Extra facts: 6000 people, 4500 children
The tickets cost Mr. Stone $22.50.

1. While grocery shopping, Mrs. Olsen found 375 g and 525 g boxes of cornflakes. She also saw 400 g and 600 g boxes of bran cereal. Mrs. Olsen decided to buy a large box of cornflakes and the small box of bran flakes. How much more or less than a kilogram of cereal did Mrs. Olsen buy?

2. Mr. Burns drove from his home to the lake at the speed of 50 miles per hour. He left at 8 o'clock in the morning and drove the distance of 125 miles. How long did it take Mr. Burns to drive the distance to the lake?

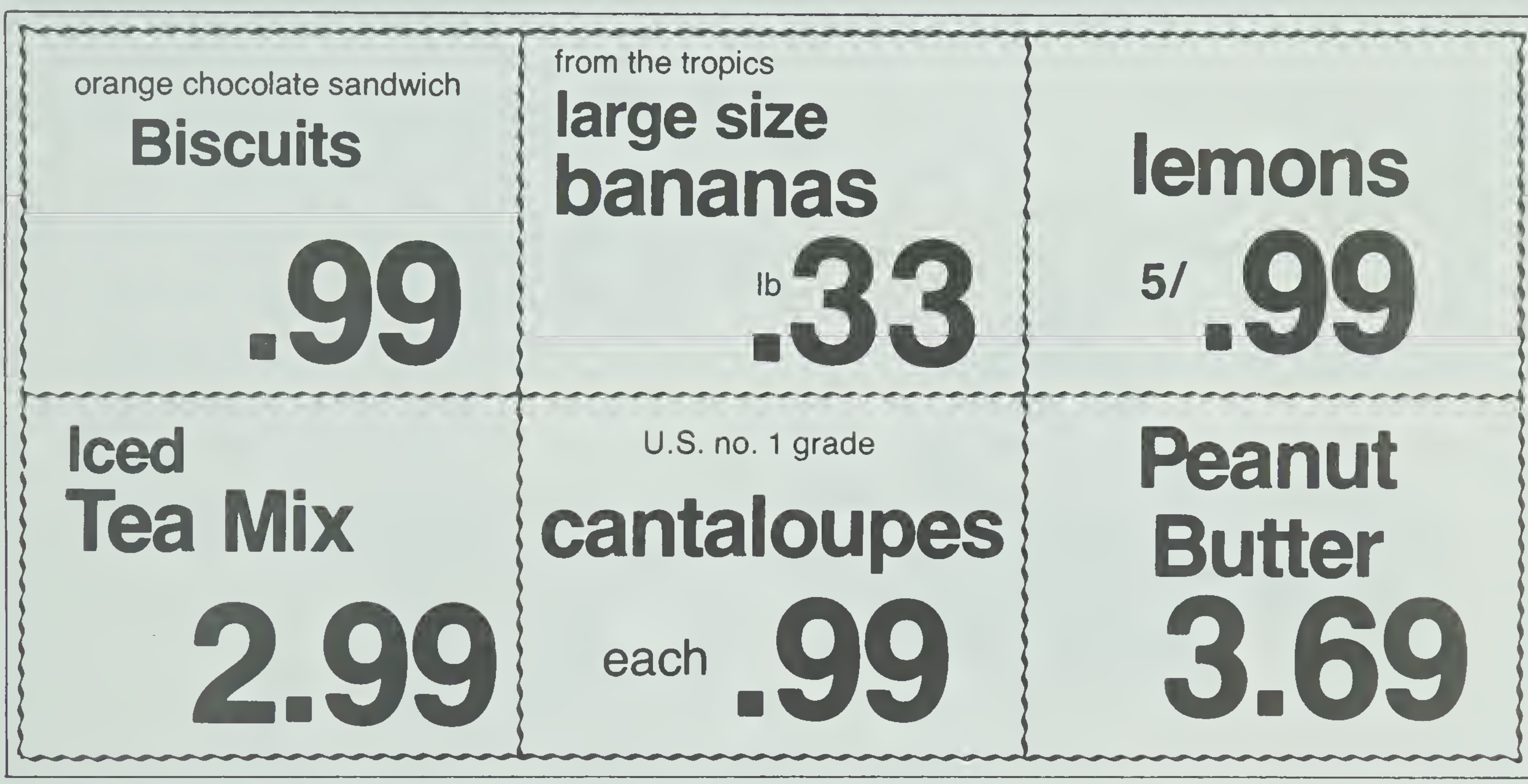
3. In one season, a football team scored 315 points. They won the play-off game by 3 points. There were 79 points scored in the game. What was the score of the game?

4. About how many times would the area of the smallest Great Lake fit into the area of the largest Great Lake?

5. How much less area is covered by Lake Michigan than Lake Superior?

6. The Great Slave Lake of the Northwest Territories has an area of 29,000 km². Which of the Great Lakes have smaller areas than Great Slave Lake?

Lake	Area (km²)
Superior	82,000
Huron	60,000
Michigan	58,000
Erie	26,000
Ontario	20,000



7. Mr. Elliot bought 2 pounds of bananas, 3 cantaloupes, and 1 jar of peanut butter. He gave the clerk a ten-dollar bill. What was his change?

8. Mrs. Piper bought one of every item which did not have to be weighed. How much did Mrs. Piper spend? Round each price up to the nearest cent.

9. What is the cost of half a pound of bananas and 2 packages of biscuits? Round up to the nearest cent.

Sometimes not enough facts are given to solve a problem.

Problem:

Sheila gets an allowance of $3/week and she earns extra money by babysitting. She wants to buy a camera which costs $49.95. She has already saved $22.75 for the camera. If she saves all her money each week, how many weeks will it be before she has enough to buy the camera?

Money still needed: $49.95 − $22.75 = $27.20

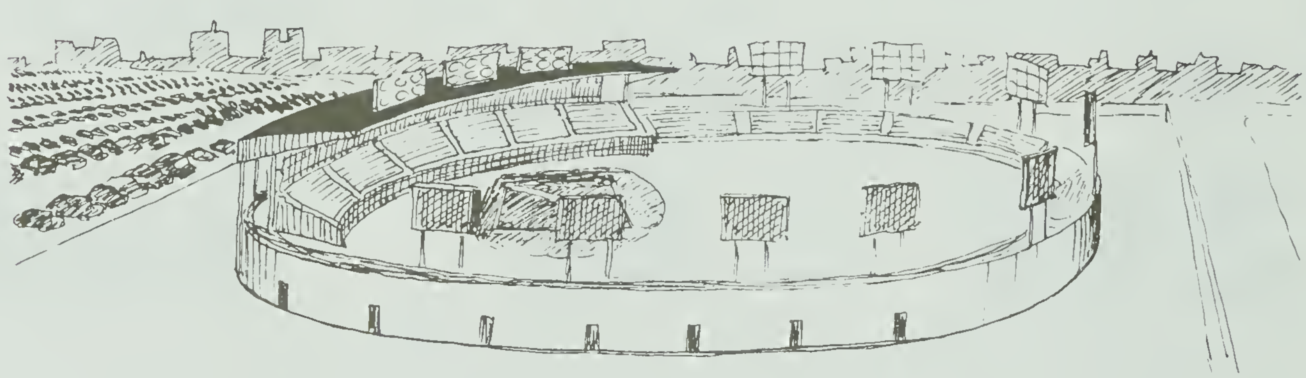
Ways Sheila earns money:

1. $3/week allowance
2. babysitting

Missing fact: How much does Sheila earn by babysitting?

If the problem cannot be solved, write which facts are missing.

1. At a major league baseball game, 11,463 fans bought tickets that cost $7.50. There were 13,512 tickets sold at $5.50 each. The rest of the fans bought general admission tickets at $2.50 each. How much money was taken in from ticket sales?



2. How much longer is the Mackenzie River than the Colorado River?

3. How much longer is the Nile River than the Mackenzie River?

4. Is the Mackenzie River the longest river in North America?

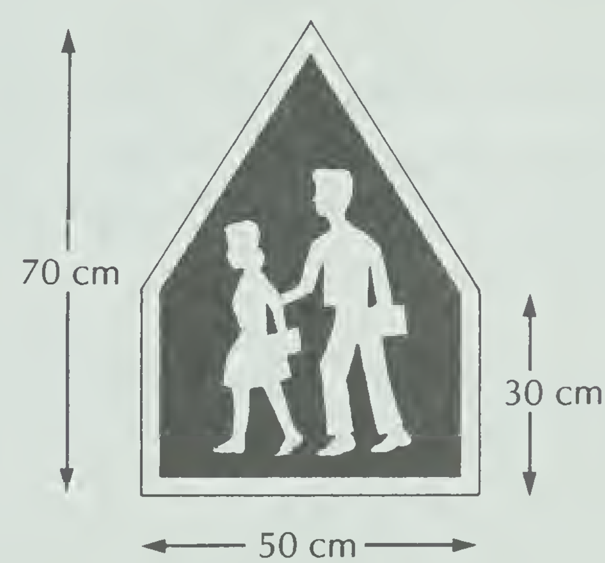
River	Length
Colorado	1450 mi
Mackenzie	2540 mi
Ottawa	690 mi
South Pacific	440 mi
Snake	1000 mi
Yukon	1980 mi

5. At 7 o'clock in the morning, the Martels started their trip to their summer cottage which was 280 miles away. Mr. Martel stopped 10 min to fill his tank with 12 gallons of gas. Later they stopped 50 min for lunch. The speed limit on expressways is 55 miles per hour. For how long did the Martels stop during their trip?

6. Great Slave Lake in the Northwest Territories has an area of 11,000 sq miles. Lake Michigan has an area of 22,300 sq miles. The average depth of Great Slave Lake is 650 feet. How much deeper is Great Slave Lake than Lake Michigan?

7. A popular magazine sells for $1.75 at the newsstands. The publishers have a special subscription rate of $0.94 per issue for 26 weeks, or $0.89 per issue for 52 weeks, or $0.85 per issue for a 2-year subscription. How much will Mr. Perkins save if he takes out a subscription for the magazine?

8. Motorists are warned about school zones with signs like the figure shown. What is the perimeter of the sign?



9. When Chris takes a bath, he uses about 35 gallons of water. When he takes a shower, he uses about 2 gallons a minute. How much water does Chris save when he takes a shower instead of a bath?

10. Mr. and Mrs. Hoyt are trying to decide on a new rug for their living room which is 6 yards long and 5 yards wide. They like a brown rug at $17.95 per square yard, a beige rug at $19.95 per square yard, and a flower print rug at $24.95 per square yard. How much would they spend if they bought the beige rug?

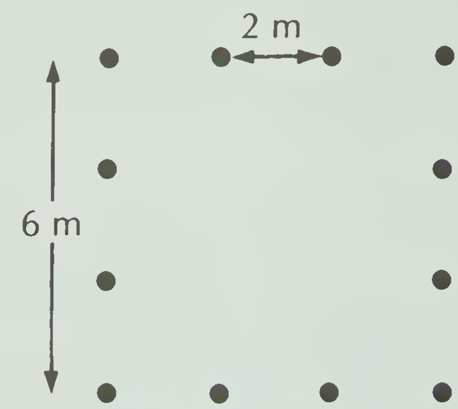
UNIT 5 Using Diagrams

A diagram can be used to understand and solve a problem.

Problem:
Mr. Hart wants to fence in a square garden that is 6 m on each side. He wants to place the fence posts 2 m apart. How many posts will he need?

Solution:
The diagram of the garden shows the posts as 2 m apart. There are 4 posts on each side, but only $(4 \times 4) - 4$ or 12 posts in all.

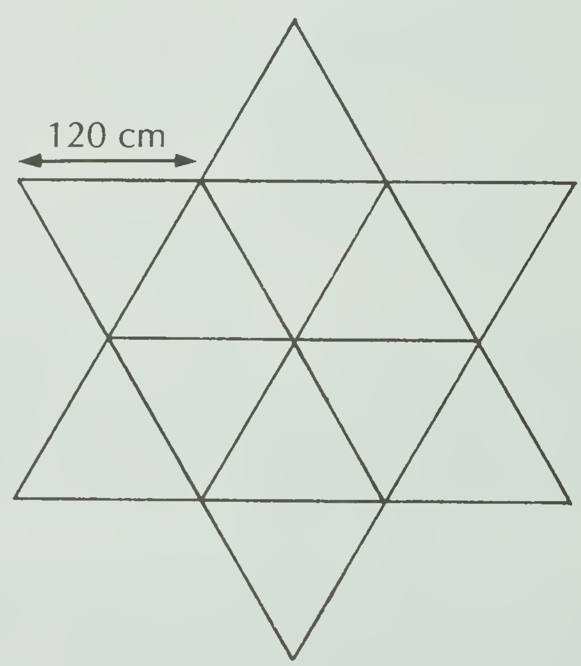
We subtract 4 because each corner post is on 2 sides.



Use a diagram to help you solve the problem.

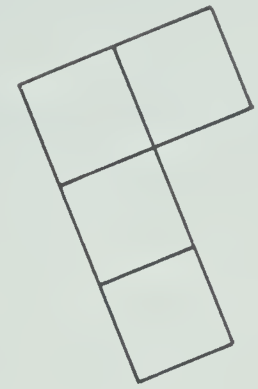
1. Mrs. Quinn has a rose garden in the shape of a triangle. Each side of the triangle has a length of 8 m. On each side of the garden, a rose bush has been planted every half meter. How many rose bushes are on the sides of the garden?

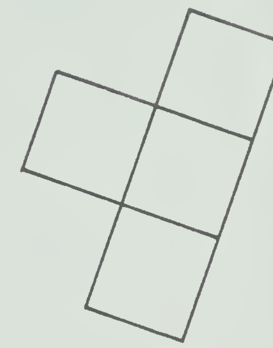
2. One of the gardens in a park has the shape at the right with a differently colored flower planted in each section. At every point where two or more lines meet, there is an automatic sprinkler valve. How many valves are there in the garden?



3. A rabbit eats his way completely around the edge of the garden. If a side of each star point is 120 cm long, how far does the rabbit travel?

4. Four square tiles can be put together to form various figures. Two of those figures are shown at the right. Draw the three other figures that can be made with four square tiles. Any two tiles must have at least one edge in common.

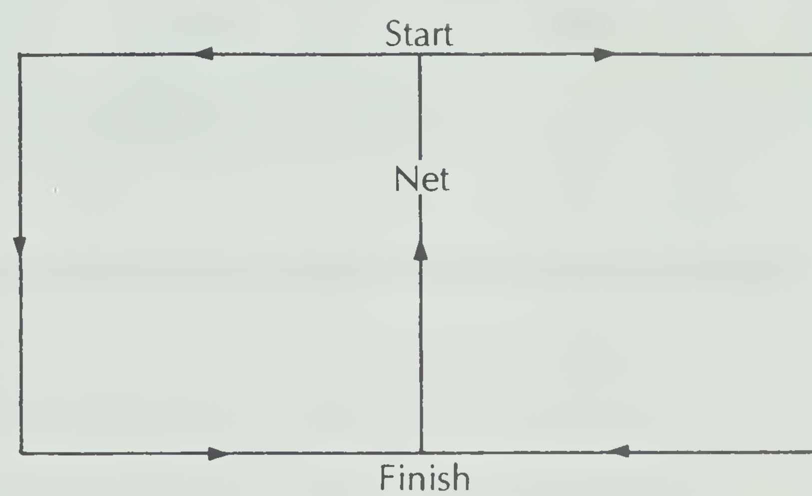




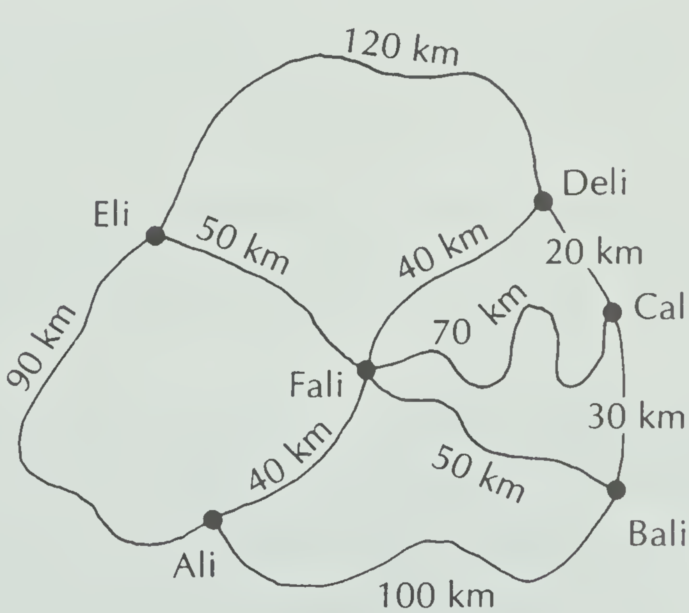
5. Suppose the tiles in the previous problem have sides measuring 2.5 cm. Which of the five figures made with four tiles has the smallest perimeter?

6. The wall of a living room is 5 m long and 3 m high. The wall has a large picture window 3 m long and 2 m high. How many square meters of wallpaper would cover the wall?

7. An ant travels around the edge of a ping pong table and goes under the net without retracing any steps. One possible path is shown in the diagram. How far does the ant travel if the table is 304 cm long and 137 cm wide?



8. Country roads connect 6 towns. You want to visit all the towns starting from Ali. Find the path of the shortest distance that does so.

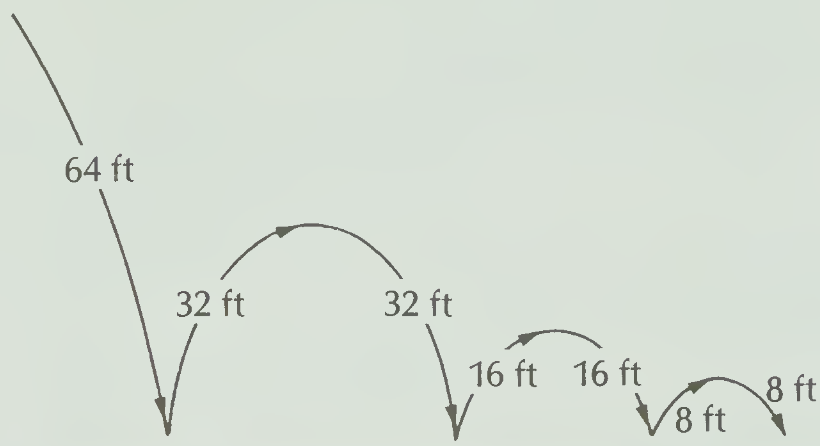


Problem:

A ball is dropped from a height of 64 ft. It bounces upward 32 ft, and each time it returns upward it goes half as high as on the previous bounce. How far does the ball travel after it has touched the ground four times?

Solution:

Draw a diagram to picture the ball's upward and downward path.



The distance of 64 ft is covered only once. The succeeding distances are covered twice, once on the way up and once on the way down.

$64 + 32 + 32 + 16 + 16 + 8 + 8 = 176$

The total distance travelled by the ball is 176 ft.

Draw a diagram to help you solve the problem.

1. Blackstone is 100 km from Kingston. Wheelhouse is midway between Blackstone and Kingston. A tunnel, on the same road as the three towns, is 30 km from Kingston. How far is the tunnel from Wheelhouse?

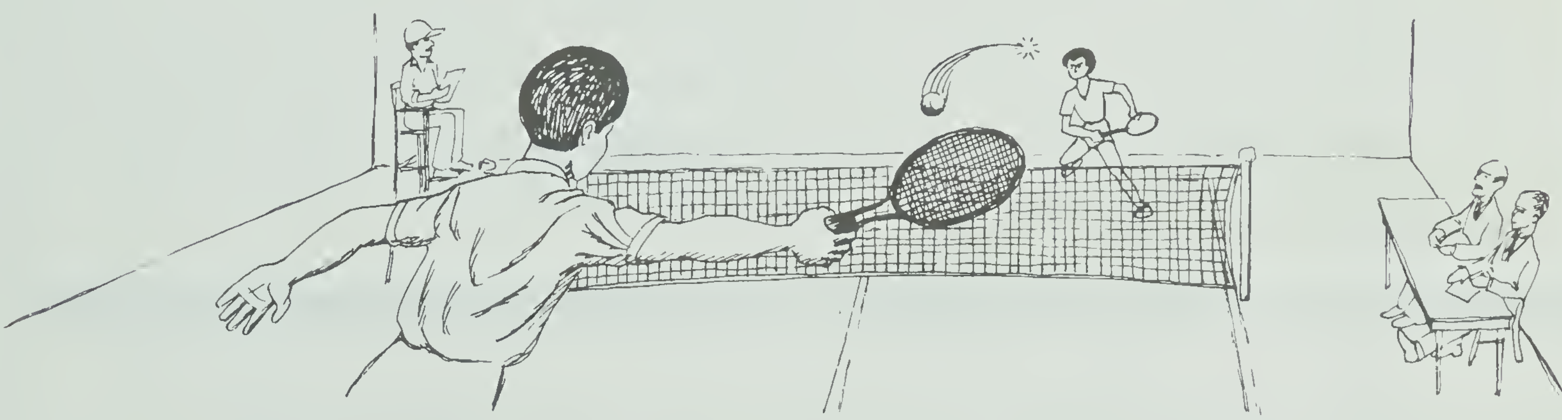
2. The center section of a recreation hall is to be tiled. The hall is 65 ft long and 40 ft wide. The area to be tiled is 50 ft by 30 ft. What is the area of the part of the floor which will *not* be tiled?

3. Sara trades in her animals for smaller animals always getting 3 new animals for each animal she trades in. She traded in her horse for 3 cows, each cow for 3 pigs, and each pig for 3 goats. If Sara traded the goats for lambs and the lambs for ducks, how many ducks did she finish up with?

4. From Sturbridge to Dilburg, there are 3 roads. From Dilburg to Briarpatch, there are 4 roads; and from Briarpatch to Coaltown, there are 2 roads. If you plan a trip from Sturbridge to Coaltown, how many different routes could you take?

5. A grocer makes a display of cans of peaches by stacking them so that there is 1 can on top, 2 cans in the second row from the top, 3 cans in the next row, and so on until he has 20 rows of cans. How many cans of peaches will there be in the stack altogether?

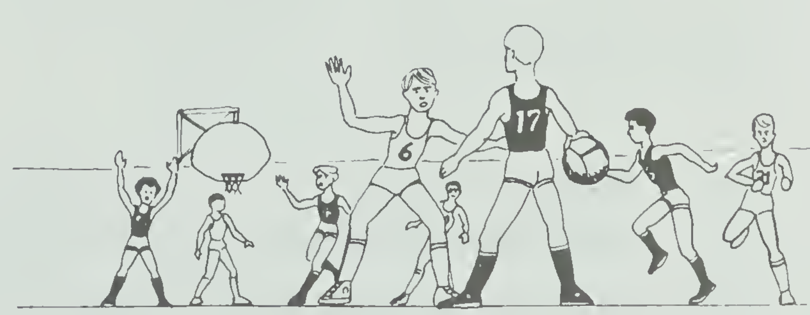
6. In a single elimination tennis tournament, Irene beat Steve, but lost to Ian. Joe beat Ann, but lost to Beth. Dina lost to Ian. Beth beat Ted and Joe, but lost to Ian. Who won the tournament?



7. Seventeen students ordered hot dogs and fries. Eleven students ordered fries. Twelve ordered hot dogs. Six students ordered both fries and hot dogs. How many students ordered only hot dogs?

8. Of the 20 cars in the school parking lot, 4 are silver and have 2 doors; 18 have 2 doors; and 6 are silver. How many cars are silver but do not have 2 doors?

9. In a single elimination basketball championship, the Bears lost to the Badgers who beat the Beavers. The Bisons lost to the Beavers. Which team won the championship?



UNIT 6 Using Equations

Equations help you guess and test possible solutions to a problem.

Problem:
If 6 bananas cost 90¢, what does one banana cost?

Solution:
(1) Let the cost of a banana be represented by *B*.
(2) Write an equation: $6 \times B = 90$.
(3) Guess and test what the unknown is.

1st guess:	*2nd guess:*
One banana costs 12¢.	One banana costs 15¢.
$6 \times \boxed{B} = 90$	$6 \times \boxed{B} = 90$
$6 \times \boxed{12} = 72$	$6 \times \boxed{15} = 90$
Too small!	**Just right!**

One banana costs 15¢.

Choose a letter to represent what is unknown. Then write an equation and guess and test the solution.

1. If 3 footballs cost $51.00, what does one football cost?

2. Zeke needs 34 more baseball cards to bring his collection up to 200. How many baseball cards does he have now?

3. Mike stood on a scale and saw that he had a mass of 52 kg. Then Ross got on the scale and the two boys together saw a reading of 99 kg. What is Ross's mass?

4. The Hootons covered their entire living room floor with a new rug. The rug cost $18.00/m and the Hootons paid a total of $360.00 for the rug. What is the area of their living room?

5. Lisa went to the movies with a ten-dollar bill and came home with $4.78. How much did she spend?

6. On a tennis tour, Chris Evert Lloyd played 108 games and won 83 of them. How many games did she lose?

7. Angie played the video game Pac-Man for 5 min. After 1 min, Pac-Man had eaten 48 video dots. When Angie was finished, Pac-Man had eaten a total of 263 dots. How many dots were eaten by Pac-Man in the last 4 min?

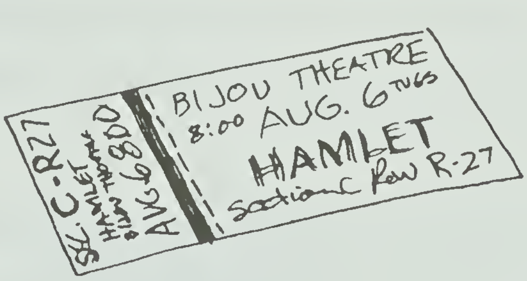
8. After doing the dinner dishes, Sparky said, "I must have washed a million dishes!" Actually, there was a total of 137 dishes. How many more dishes would Sparky have to wash to bring his total to a million?

9. In Glenda's book, there are about 300 words on each page. There are about 9 words on a line. About how many lines are there on a page?

10. Ursula bought a sheet containing 25 postage stamps. The cost of the sheet was $8.75. What was the price of each stamp?

11. Alex can run from his home to the park and back in 22 min. His older brother can run the same distance in 17 min. By how many minutes would Alex have to improve his running time to equal that of his brother?

12. Mr. Naber bought 5 theatre tickets for $42.50. What was the price per ticket?



13. The Earlyville Eagles played 120 softball games last season and lost 53 of them. How many games did they win?

Sometimes two equations are used to help you guess and test possible solutions to a problem.

Problem:

If you add Maria's age to her father's age, you get 36.
If you subtract Maria's age from her father's age, you get 20. What are the ages of Maria and her father?

Solution:

(1) Let Maria's age be M. Let her father's age be F.

(2) Write two equations: $F + M = 36$ and $F - M = 20$.

(3) Guess and test to find the 2 unknowns.

1st guess:	*2nd guess:*
Maria is 9.	Maria is 8.
Her father is 27.	Her father is 28.
$F + M = 36$	$F + M = 36$
$27 + 9 = 36$	$28 + 8 = 36$
$F - M = 20$	$F - M = 20$
$27 - 9 = 18$	$28 - 8 = 20$
Not quite!	**Just right!**

Write two equations. Then guess and test the solution.

1. If you add two numbers, the sum is 15. If you multiply the two numbers, the product is 56. What are the numbers?

2. A chicken and an egg cost \$2.00. The chicken costs \$1.60 more than the egg. What is the cost of the chicken?

3. In the baseball game between the Toads and the Frogs, there were 46 runs scored. The Toads scored 8 more runs than the Frogs. What was the score of the game?

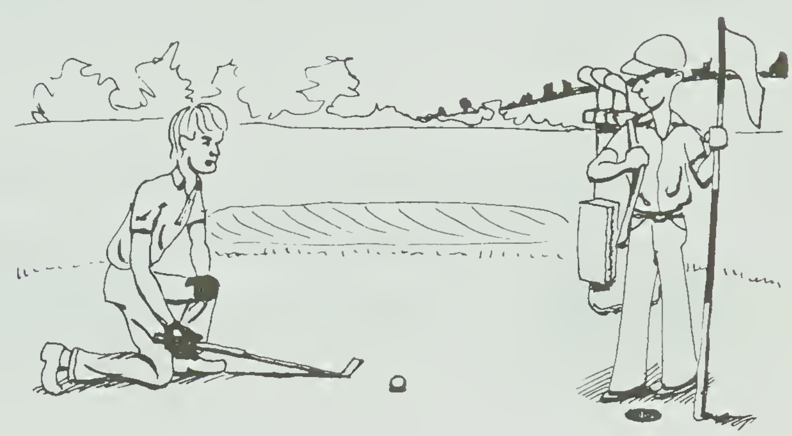
4. A dress and a hat cost $40.00. The dress cost $6.00 more than the hat. What was the cost of the hat?

5. When Paul added his height to his father's height, he got 352 cm. When Paul subtracted his height from his father's height, he got 18 cm. What is Paul's height?

6. At the rock concert, there were 8400 people. There were 600 more females than males. How many females were at the concert?

7. Together Timmy and Eddie weigh 205 pounds. Timmy weighs 33 pounds more than Eddie. What is each boy's weight?

8. A champion golfer had an 18-hole score of 68. He took 6 strokes less on the back 9 holes than on the front 9 holes. How much did he score on the front 9 holes and the back 9 holes?



9. A rectangle has a perimeter of 26 inches and an area of 40 square inches. What are the length and width of the rectangle?

10. In a field, the number of pigs and geese is 20. The number of legs is 70. How many pigs and how many geese are in the field?

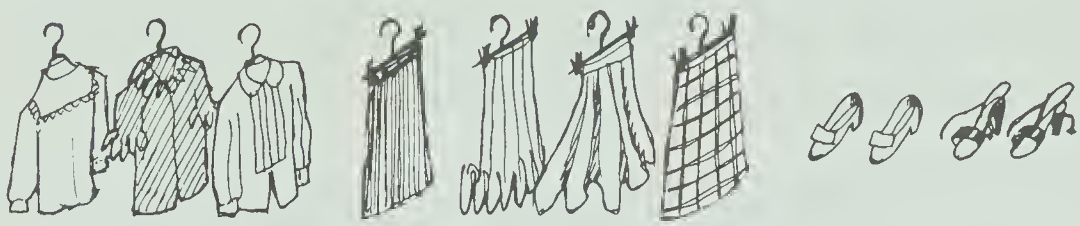
11. Together, the two cars in the Healy family have been driven for 52,000 miles. Their older car has been driven for 12,000 miles more than their newer car. For how many miles has each car been driven?

12. Tom and Anna each keep guppies in their aquariums. In all, they both have 65 guppies. Tom has 9 fewer than Anna. How many guppies does Tom have?

UNIT 7 Review Problems

Solve each problem using strategies you have already practiced.

1. Meg has a choice of 3 blouses, 4 skirts, and 2 pairs of shoes as she plans her outfit for a school dance. How many different outfits are possible?



2. Zelda bought 5 tickets for a hockey game. The tickets were numbered in order. Zelda added the numbers and got 4165. What were the ticket numbers?

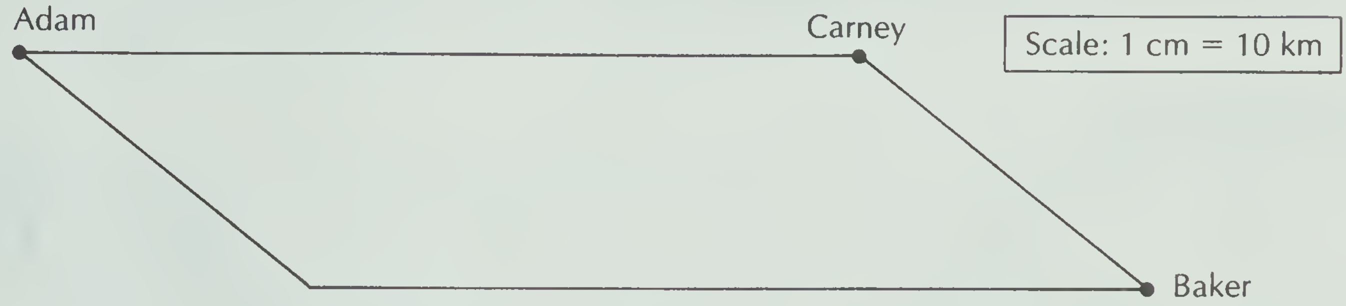
3. In Apple Blossom School, there are 18 times as many students as there are adults. Altogether there are 399 persons at the school. How many students are at Apple Blossom School?

4. At the Fish-Fry Shop, one take-out fish dinner costs \$4.50. If a second dinner is bought, it costs \$4.00. For each additional take-out dinner, the price is reduced by 50¢. If the Hurst family buys 6 take-out dinners, what will be the cost?

5. How many dinners from the Fish-Fry Shop would have to be ordered before you could get a free dinner?

6. Suppose that if tickets for a concert are bought 8 weeks in advance, they cost \$5.00 each; 7 weeks in advance, \$10 each; 6 weeks in advance, \$15 each; and so on. How much would a ticket cost if it were bought on the night of the concert?

7. The map below shows the roads joining three towns. Every centimeter on the map represents 10 km. What is the actual distance from Adam to Baker to Carney and back to Adam?



8. At noon, the ball park was empty. At 12:01, there were 10 people in the park; at 12:02, 20 people; at 12:04, 80 people; and so on. If this pattern continues, how many people will be in the park at 12:10?

9. If there are 40,000 seats in the ballpark, at what time will the park be full?

10. Mr. Watkins earns $12.00/h for 40 h/week. If he works more than 40 h, Mr. Watkins gets time and a half or $18.00/h. How much does Mr. Watkins earn in one week?

Sales and Profits of Canada's Three Largest Grain Cooperatives for One Year

Name of Cooperative	Sales	Profits
Saskatchewan Wheat	$1,941,433,000	$72,744
Alberta Wheat	$1,390,978,000	$34,592
United Grain	$1,374,576,000	$ 7,603

11. How much greater were the sales of Alberta Wheat than United Grain?

12. If profits remained about the same every year, about how many years would it take United Grain to have about the same profit that Saskatchewan Wheat makes in one year?

13. How many persons are employed by Saskatchewan Wheat?

14. Mrs. Paquette drove 35 miles to a meeting which lasted for 2 hours and 30 minutes. Then she had a lunch break for 1½ hours. The lunch bill for 6 women amounted to $44.52. If the women divided the cost equally, how much did Mrs. Paquette pay?

15. In the first two weeks of its showing, *Superman II* took in $42,500,000 in ticket sales. It was shown in 1408 theatres. *E.T.* was shown in 1116 theatres during its first two weeks and took in $44,800,000. How much more money was taken in by *E.T.*?

16. What is the fewest number of coins you can have that will pay the exact price for any amount from 1¢ to 20¢?

17. The perimeter of a photograph is 36 inches. The length is twice as long as the width. What are the length and the width of the photograph?

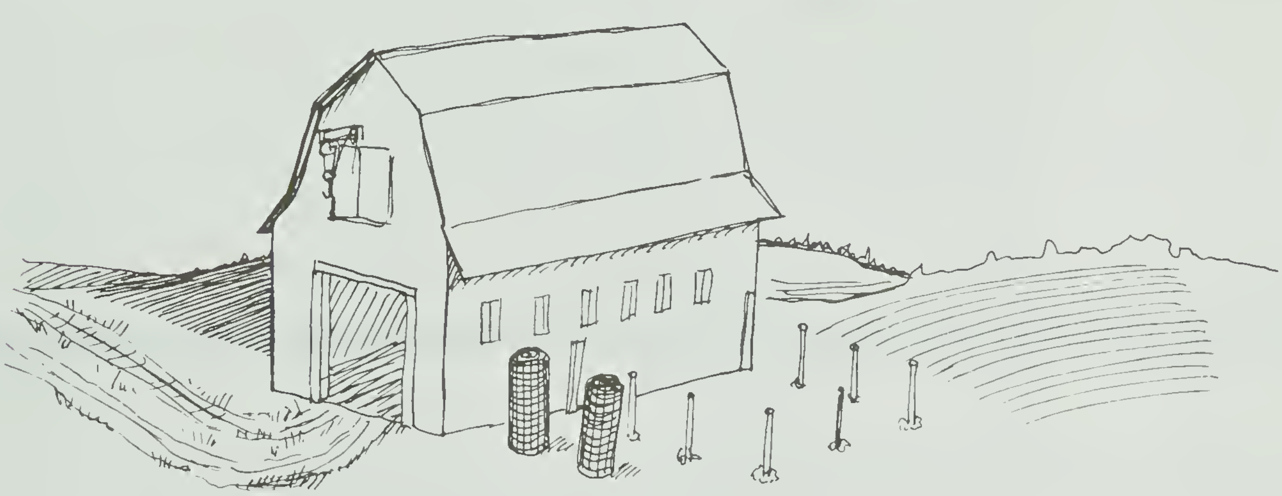
18. Mr. Tompkins, a physical education instructor, bought 4 soccer balls and 5 baseballs for the school. The total cost was $60.00. A soccer ball costs $6.00 more than a baseball. What is the price of a baseball and a soccer ball?

19. How many different rectangles with dimensions in whole numbers have a perimeter of 24?

20. Of all the rectangles with a perimeter of 24, which rectangle has the greatest area?

21. Myra's book on space adventures has 85 pages. Some pages have printing only and the rest have both printing and pictures. There are about 200 words on a printed page and about 50 words on a pictured page. The book has about 11,000 words in all. How many of the pages have pictures?

22. A farmer wishes to put up a fence to make a rectangular pig pen. One side of the pen will be his barn, so the farmer needs fencing on only three sides. He has 100 feet of fencing and would like to make the pig pen with as large an area as possible. What should the dimensions of the pig pen be?



23. The combined populations of Mayville and Branby is 5200 people. Branby has 800 more people than Mayville. How many people live in Mayville?

24. The sum of three consecutive numbers is 276. What are the three numbers?

25. Multiply the number, 11, by the numbers from 10 to 25. Study the products. What pattern do you notice?

UNIT 8 Organized Lists

Some problems can be solved more easily when facts are organized in a list.

Problem:
The average annual rainfall for some cities in the United States was found to be: Albuquerque, 8.3 inches; Boston, 41.6 inches; Cheyenne, 14.5 inches; Des Moines, 31.5 inches; Mobile, 63.3 inches; Phoenix, 7.4 inches; Seattle, 40.3 inches. Which parts of the U.S. appear to have the most rainfall?

Solution:
Make a list of the amounts of rainfall in increasing order.

City	*Rain*
Phoenix	7.4 inches
Albuquerque	8.3 inches
Cheyenne	14.5 inches
Des Moines	31.5 inches
Seattle	40.3 inches
Boston	41.6 inches
Mobile	63.3 inches

The coastal regions of the United States usually have more rainfall.

Make a list to help you answer the questions.

1. The greatest depths of some of the world's largest seas are the Caribbean Sea, 7540 m; the Mediterranean Sea, 5153 m; the Sea of Okhotsk, 3376 m; Hudson Bay, 258 m; North Sea, 671 m; Black Sea, 2246 m; Red Sea, 2196 m. Are the following statements true or false?
   a. The deepest part of Hudson Bay is about one tenth the deepest part of the Caribbean Sea.

   b. The deepest part of the Black Sea is about 100 m deeper than the deepest part of the Red Sea.

   c. The deepest part of the Mediterranean Sea is over 5 km deep.

2. The greatest life span of some animals are the giant tortoise, 190 years; the elephant, 84 years; the eagle, 55 years; the gorilla, 33 years; the tiger, 25 years; the rattlesnake, 20 years; and the pig, 10 years.

   a. Is the greatest life span of an elephant more or less than half the greatest life span of a giant tortoise?

   b. The greatest life span of which animal is twice as much as that of the pig?

   c. Do the larger, heavier animals have the greatest life spans?

   d. About how many generations of gorillas could occur in 100 years?

3.

Six Planets Nearest the Sun			
Planet	Distance From Sun (Earth = 1 unit)	Time To Go Around Sun	Diameter (Earth = 1 unit)
Mercury	0.39	88.0 days	0.38
Venus	0.72	224.7 days	0.82
Earth	1.00	365.2 days	1.00
Mars	1.52	1.9 years	0.53
Jupiter	5.20	11.9 years	11.20
Saturn	9.54	29.5 years	9.36

Are the following statements true or false?

   a. The farther away from the sun, the longer it takes to go around the sun.

   b. The farther away from the sun, the bigger the planet.

   c. In the time it takes the earth to go around the sun, Mercury circles the sun about four times.

   d. It takes Jupiter about six times as long to circle the sun as it takes Mars.

   e. The diameter of Mars is about half the diameter of Earth, and it takes Mars about half as long to circle the Sun as it takes the Earth.

Problem:

A bell rings every 2 min, a horn blows every 3 min, and a rooster crows every 4 min. At noon they all sound off together. In how many minutes will all three sound off together again?

Solution:

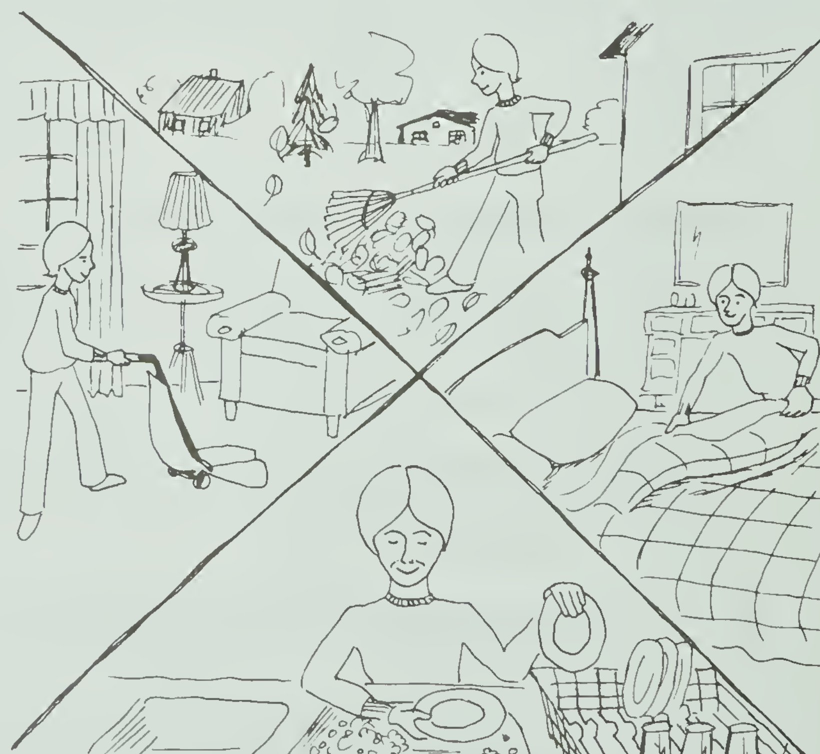
List the times when all three sounds are made.

Bell	12:02	12:04	12:06	12:08	12:10	12:12	12:14
Horn	12:03	12:06	12:09	12:12	12:15	12:18	12:21
Rooster	12:04	12:08	12:12	12:16	12:20	12:24	12:28

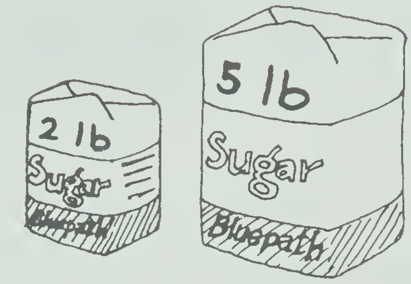
At 12:12, all three will sound off again.

Make a list to help you solve the problem.

1. Ed jogs every 3rd day, Ned jogs every 4th day, and Fred jogs every 6th day. If they all jogged today, when will they all jog on the same day again?
2. Tony lined up his toy soldiers by 6s and there were none left over. He lined them up again by 8s and there were none left over. He tried to line them up again by 10s, but there were 2 left over. What is the smallest number of toy soldiers which Tony could have?
3. Rosie promised to do chores for 2 weeks. Her mother offered to pay her 1¢ the first day, 2¢ the second day, 4¢ the third day, and double the amount of the preceding day for 2 weeks. How much money would Rosie make?
4. Rosie's father offered her $1.00 the first day, $2.00 the second day, $3.00 the third day, and one more dollar than the previous day for 2 weeks. Is Rosie's father's offer better?



5. How many ways could you obtain 40 lb of sugar using only 2 lb or 5 lb bags?



6. How many different pairs of numbers have a sum of 100 and a digit sum of 19?

   Hint: $85 + 15 = 100$ $(8 + 5) + (1 + 5) = 19$

7. How many whole numbers between 1 and 1000 are evenly divisible by 2, 3, and 5?

   Hint: $60 \div 2 = 30$ $60 \div 3 = 20$ $60 \div 5 = 12$

8. How many times would the digit, 2, be written if you wrote down all the whole numbers from 1 to 100?

9. Mr. Francona wants to buy 30 pounds of nails. Some of the nails come in 3 pound boxes, the rest in 2 pound boxes. How many ways could he purchase the nails?

10. The Downsview Diner has three kinds of burgers: taco burger, pizza burger, and limburger; two kinds of potatoes: fries or hash browns; and four different drinks: chocolate, cola, orange, or tea. If you order one burger, one kind of potato, and one drink, how many different combinations are possible?

UNIT 9 Ratio, Proportion, and Percent

Ratios and proportions can help you solve problems.

Problem:

For a barbecue party, Mrs. Tully estimated that 2 chickens would serve 5 people. How many chickens will Mrs. Tully need to serve 40 people?

Solution:

One method

Make a table. The numbers of chicken and people increase proportionately.

Chickens needed	2	4	8	16
Number of people	5	10	20	40

Another method

Write a proportion showing the relationship of chickens to people.

$$\frac{\text{chickens}}{\text{people}} \qquad \frac{2}{5} = \frac{N}{40} \qquad\qquad \frac{2}{5} \overset{\boxed{\times 8}}{=} \frac{16}{40}$$

Mrs. Tully needs 16 chickens for 40 people.

Make a table or write a proportion to solve the problem.

1. Lynn had 3 out of the first 5 problems on a math test solved correctly. If she keeps up the same rate of success for the entire test, how many correct answers will she have out of 30 problems?

2. When Drew plays golf, he loses a golf tee at every one of the 18 holes. If golf tees cost 2 for 15¢, what do tees cost Drew when he completes 18 holes of golf?

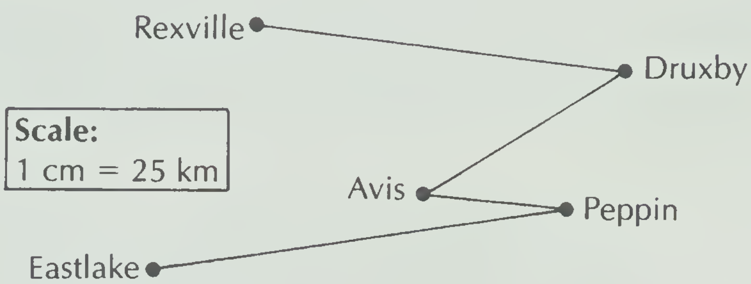
3. Cat food was on special at 2 cans for 59¢. Bruce stocked up with 32 cans. What was the cost?

4. A man and a wife are walking together. The man takes two steps for every three steps taken by his wife. How many steps will be taken by the wife if the man takes 50 steps?

5. Ray did 400 sit-ups in 8 min. What was his rate of sit-ups per minute?

6. What is the actual distance between:

   a. Rexville and Peppin?
   b. Avis and Eastlake?
   c. Rexville and Eastlake?

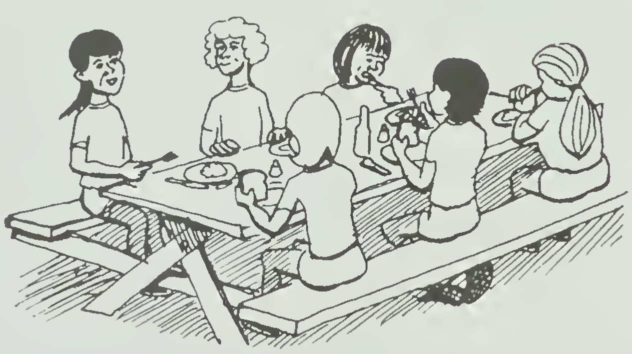


7. Lois estimates that 2 large pizzas will serve 10 people and that 5 small pizzas will also serve 10 people. How many large pizzas will serve 40 people?

8. In baseball, an Earned Run Average (ERA) is the average number of runs allowed by a pitcher per 9 inning game. Hubert Leonard of the Boston Red Sox established the record low ERA of 1.00 runs per game in 1914. Leonard pitched a total of 225 innings that season. How many runs did he allow that year?

9. At Rhonda's birthday party, 14 L of pop were drunk. If Rhonda allowed 2 L for every 5 persons, how many people were at her party?

10. At Camp Lakeshore, 6 children eat at one lunch table. If there are 90 children at the camp, how many tables are needed?



11. A professional photographer does not use 2 out of every 3 of the pictures he takes. If he throws out 48 pictures, how many has he taken?

Proportions can help you solve percent problems.

Problem:

The marketers of *Brite* toothpaste claim that 3 out of 5 people use their product. What percent of people is that?

Solution:

One method

Make a table. Increase the ratio, 3 to 5, proportionately until the number *per 100* is reached.

People using *Brite*	3	6	60
Total Number of people	5	10	100

Another method

Write a proportion showing the ratio, 3 to 5, as equal to a number *per 100.*

$$\frac{3}{5} = \frac{N}{100} \qquad \frac{3}{5} \underset{\boxed{\times 20}}{\overset{\boxed{\times 20}}{=}} \frac{60}{100}$$

The marketers claim that 60 out of 100, or 60%, of the people use *Brite*.

Solve the problem using a table or a proportion.

1. In a survey, 7 out of 20 people said they disliked television football. If this survey was accurate, what percent of people dislike television football?

2. Sharon is reading a 300-page book. She has already read 180 pages. What percent of the book has she read?

3. In a basketball shooting match, Trish made 13 baskets out of 25 shots. What was her shooting percentage?

4. Beryl wants to buy a new bike which costs \$150.00. There is a tax of 6% on the selling price. What is the amount of tax?

5. Joanne's mother sold a house for her client for \$80,000.00 She received a commission of 5% of the selling price. How much did Joanne's mother earn on the sale?

6. A television set with an original price of \$600.00 was marked down at a 20% discount. How much less will be paid by the buyer at the sale price?

7. A 500 g jar of strawberry jam is 18% sugar. How many grams of sugar are in the jam?

8. Savings Bonds were being sold with an interest rate of 8%. If Mr. Bixby bought \$5000.00 worth of bonds, how much interest will he earn in the first year?

9. During a long drought, the amount of water in a city reservoir fell by 24%. If the reservoir contained 12,000,000 kL of water before the drought, how much water was left?

10. At a university, the tuition cost last year was \$1200.00. This year, the cost increased by 20%. What is the tuition cost this year?

11. Janet had a score of 85% on a social studies test. If the test had 20 questions, how many of her answers were incorrect?

12. A \$200 coat was sold for 30% less at a spring sale. What was the amount of discount on the coat?

UNIT 10 Multi-Step Problems

The solution of a problem often requires more than one step or operation.

Problem:
In a peach orchard, there are 42 rows of peach trees with 31 peach trees in each row. If each peach tree produces about 180 pounds of peaches, how many pounds of peaches should be harvested?

Solution:
Step 1
Multiply to find the total number of peach trees in the orchard.
$42 \times 31 = 1302$

Step 2
Multiply to find the number of pounds of peaches to be harvested.
$1302 \times 180 = 234{,}360$
The peach harvest is $(42 \times 31) \times 180$ or 234,360 pounds.

Solve.

1. Suppose that the peaches from the problem above can be sold for $0.59 per lb. If the total expenses for growing and marketing the peaches are $67,500.00, what will be the profit if all the peaches are sold?

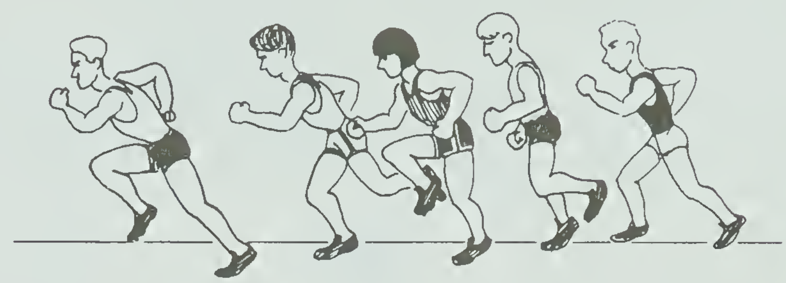
2. The students at Pinetree School are having a picnic at Crystal Lake. There will be 339 students and 13 teachers going to the picnic. Buses to carry all students and teachers can each take 44 people. How many buses will be needed?

3. A driver wishes to find how far he can go on a gallon of gas. At 43,175 miles on his odometer, he fills his tank with gas. At 43,395 miles, he fills his tank again. It takes 12 gallons to fill the tank. How many miles does he get per gallon of gas?

4. It has been estimated that on the average every person uses about 1800 gallons of water per day counting industrial use. If there are about 200 million Americans, about how many gallons of water are used in the U.S. in a year?

5. A teacher had to read 28 project reports by her students. In the first hour, she was able to read 5 of the reports. About how long will it take her to read all the reports?

6. The world's fastest humans can run 100 m in 10 s. If that speed could be maintained for 1 hour, how many kilometers would they go?



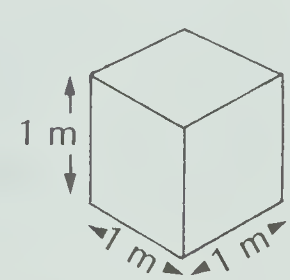
7. On five successive days, the snowfall was recorded as .8 in., .5 in., .3 in., .9 in., and 1.2 in. What was the average snowfall over the five-day period?

8. An apple has a mass of about 100 g. An orange has a mass of about 150 g. Which has the greater mass, a box of 96 oranges or a box of 140 apples?

9. Suppose an elevator has a load limit of 2000 pounds and can thereby carry about 25 children. If an average child weighs 85 pounds and an average adult weighs 150 pounds, about how many more children than adults can the elevator carry?

10. Three partners bought a piece of land for $21,000 and sold it for $36,000. How much profit did each partner make?

11. How many square centimeters would cover the entire surface of a cubic meter?



Problem:

In professional hockey, a team gets 2 points for a win and 1 point for a tie. After 40 games, the Montreal Canadians had won 21 games and tied 7 while the Boston Bruins had won 23 games and tied 4. Which team was in first place at the time if no other teams had better records?

Solution:

Step 1

Multiply and *add* to find the total points for Montreal.

$(21 \times 2) + 7 = 49$

Step 2

Multiply and *add* to find the total points for Boston.

$(23 \times 2) + 4 = 50$

Step 3

Compare: $49 < 50$.

The Boston Bruins were in first place by 1 point.

Solve.

1. In professional soccer, a team gets 6 points for a regular victory unless they win in a shootout. A shootout victory counts for 4 points. In addition, a team gets a bonus point for every goal scored up to a maximum of 3 goals per game.

   At one time during the season, the Vancouver Whitecaps had 10 regular victories and 2 shootout victories. The Seattle Sounders had 9 regular victories and 1 shootout victory. In addition, the Whitecaps had 29 bonus points, and the Sounders had 34 bonus points. Which team was ahead and by how much?

2. A Boeing 707 can carry 181 passengers; a Boeing 747 can carry 550; a McDonnell Douglas DC-10, 380; and a Lockheed L1011, 400. A 707 takes off at full capacity. Each of the other planes follow immediately with only half the passenger capacity. How many passengers are on the 4 planes?

3. Mrs. Folsom wants to rent a car for a business trip to Atlanta. The Chip-Drive Company rents its cars at $19.95 a day plus 16¢ a mile for every mile driven over 60 miles. The Clark-Car Company rents its cars for $29.95 a day with no extra charges. Mrs. Folsom estimates that her trip will last 3 days and she will drive 90 miles. Which company offers the cheaper car rental for her?

4. Len wants to go through the tunnel with his flatbed truck. He knows that the bed of the truck is 1.5 m from the ground and that his 30 cm high cartons are in stacks of 10 on the truck. Can he make it through the tunnel?

5. Shane can type 40 words a minute. An experienced secretary can type 75 words a minute. How much longer will it take Shane to type out a 3000 word essay than the secretary?

6. Early in the season, the Toronto Blue Jays had won 15 of their 25 games. In their next 25 games, the Blue Jays managed to win only 11 games. What was their winning percentage after 50 games?

7. Mr. Blanchard is paid $12 an hour plus overtime. For all hours worked over 40 in a week, he gets paid 1½ times the regular hourly rate. One week, Mr. Blanchard worked the following hours:

Day	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
Hours worked	8.5	8	9	9.5	8	6

How much did Mr. Blanchard earn that week?

UNIT 11 Making a Simpler Problem

Rounding off numbers and discarding extra facts help to make difficult-looking problems simpler.

Problem:
On a certain day, the Canadian dollar is worth 0.4529 British pounds, 0.7865 American dollars, 5.376 French francs, and 1.938 German marks. How many more francs than marks would you receive in exchange for 100 Canadian dollars?

Solution:
Make a simpler problem.
(1) Discard extra facts about British pounds and American dollars.
(2) Round 5.376 francs to 5 francs and 1.938 marks to 2 marks.
(3) Solve the simpler problem.

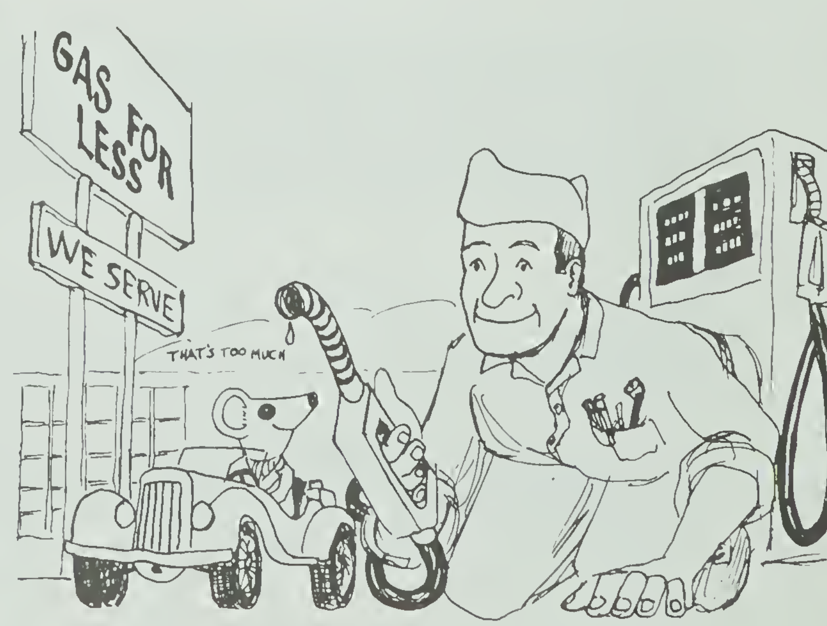
100 Canadian dollars is
$100 \times 5 = 500$ **about 500 francs.**
$100 \times 2 = 200$ **about 200 marks.**

Now solve the original problem.

100 Canadian dollars is
$100 \times 5.376 = 537.6$ francs.
$100 \times 1.938 = 193.8$ marks.

You would receive $537.6 - 193.8$, or 343.8 more francs.

Simplify the problem before solving it.



1. If one French franc is worth 0.186 American dollars, which is the better buy:

a. 1 L of gas in the U.S. at 44.9¢/L, or

b. 1 L of gas in France at 3.92 francs/L?

2. Some world records for track events are listed below.

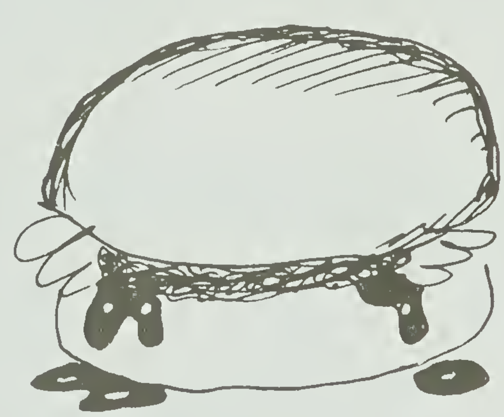
Event	*Record Holder*	*Time*
200 m	Pietró Mennea (Italy)	19.72 s
800 m	Sebastian Coe (Britain)	1 min 41.72 s
1500 m	Steve Ovett (Britain)	3 min 31.36 s

If a runner could maintain Pietro Mennea's speed for 1500 m, by how much would the 1500 m record be broken?

3. Mercury is the closest planet to the Sun and it takes only 87.97 days for Mercury to revolve around the sun. Pluto is farthest from the sun and takes 247.7 years to revolve around the sun. Jupiter is the largest planet and takes 11.86 years to revolve around the sun. About how many times does Mercury revolve around the sun while Pluto is revolving around the sun just once?

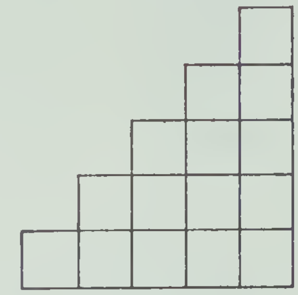
4. Gordie Howe was a 52-year-old grandfather when he was selected as an NHL All-star for a record 22nd time. He has held records for most major league hockey games played with 2421 games, most years in the NHL with 26, most career goals with 1071, most career assists with 1518, most total points with 2589, and additional playoff-game records. In his long career, about how many points did Gordie Howe have per game?

5. The largest restaurant chain in the world is McDonald's Corporation. By 1981, there were 6263 McDonald's restaurants in 26 countries. Sales in 1980 totalled $6,226,000,000. If the average customer spent $2.10, about how many customers did the McDonald's chain of restaurants have in 1980?

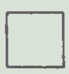


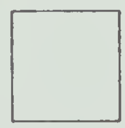
Problems can be made simpler by splitting them into smaller, sub-problems.

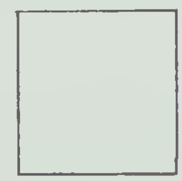
Problem:
How many squares are contained in the staircase pattern?



Solution:
Determine the different kinds of squares.


1 by 1

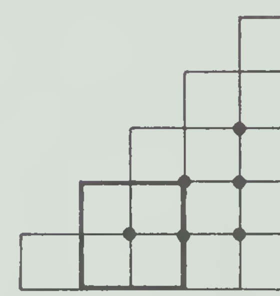

2 by 2


3 by 3

Count the number of each kind of square.

(1) 1 by 1
Counting by layers, there are 1 + 2 + 3 + 4 + 5 or 15 squares.

(2) 2 by 2
These squares overlap.
The dots show the centers of these squares.
There are 1 + 2 + 3 or 6 squares.



(3) 3 by 3
There is only one of these squares.

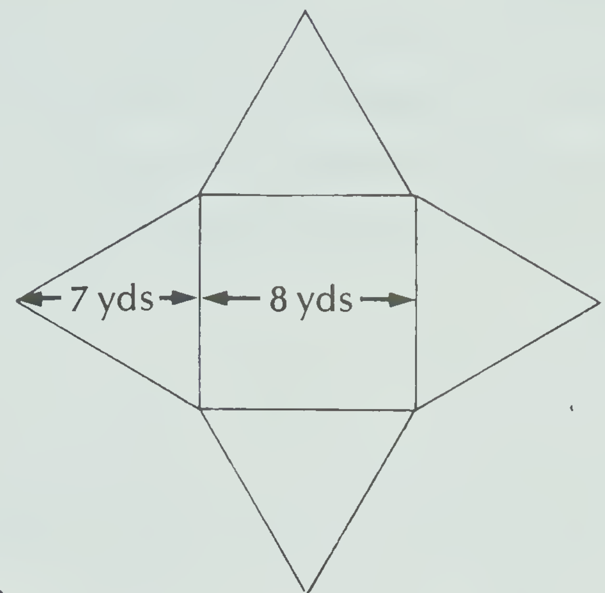
The total number of squares in the pattern is 15 + 6 + 1 = 22.

Solve.

1. How many squares are there in the checkerboard figure?

2. Babe Ruth holds the record for most total bases in a season. A single counts for 1 base, a double counts for 2 bases, a triple counts for 3 bases, and a homer counts for 4 bases. Ruth had 85 singles, 44 doubles, 16 triples, and 59 homers. How many total bases did he get?

3. A garden is in the shape of the figure at the right. It consists of a square and 4 congruent triangles. What is the area of the garden?



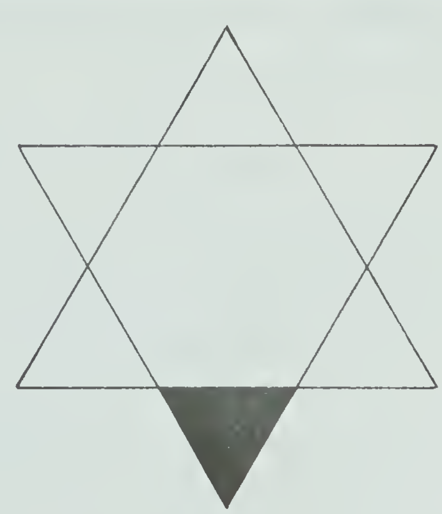
4. The perimeter of a picture frame is 140 cm. The length is 40 cm. What is the area enclosed by the frame?

5. On a math test, Karen got 7 of the first 10 problems right. There were 20 problems on the test and her total score was 80%. How many problems did she get right on the second half of the test?

6. The numerals 333, 7777, and 88 all contain repeated, single digits. How many numerals between 11 and 999,999 contain repeated, single digits?

7. Mr. McNair left at 9:00 in the morning. He drove 140 miles at 40 mph. He stopped for lunch for 45 min. Then he drove 135 miles more at 45 mph and arrived at his destination. At what time did he arrive?

8. The 6-pointed star shown at the right is formed by laying two congruent triangles upon each other to form 6 small triangles and a hexagon. If the shaded triangle has an area of 4 square miles, what is the area of the entire star?



UNIT 12 Reasonable Answers

Use common sense to decide if an answer is reasonable.

Problem:
In the 1961-62 basketball season, Wilt Chamberlain established a record for professional basketball by scoring 4029 points in 80 games. What was Wilt Chamberlain's average per game?

a. 5.4 points **b.** 50.4 points **c.** 504 points

Solution:
Common sense tells you:

5.4 points are not high enough to be a record.

504 points are too many points for one game.

50.4 points are a reasonable number of points for a game.

Wilt Chamberlain's average per game was 50.4 points.

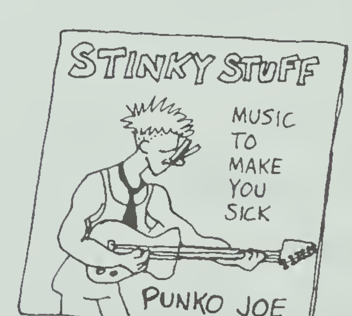
Choose a reasonable answer.

1. A hockey team averaged 4.2 goals per game during a season of 80 games. About how many goals did the team score that season?

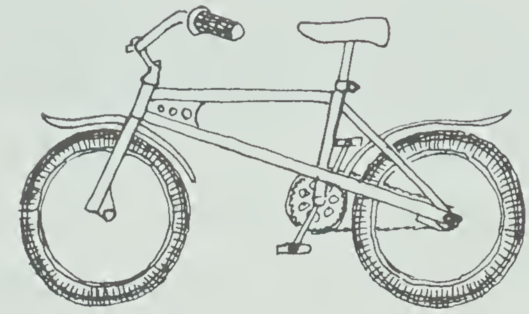
a. 34 **b.** 340 **c.** 3400

2. A rock album went on sale for $6.95. In one day, a record shop sold 116 of the albums. How much did the record shop receive in sales for that album in the one day?

a. $80.62 **b.** $806.20 **c.** $8062.00



3. The regular price on a bicycle was $160.00. The bike went on sale at a discount of 20%. What was the sale price of the bike?



a. $32.00 b. $128.00 c. $192.00

4. For a backyard barbecue, Linda expects about 40 people. She wants to have enough hamburger meat so that each person will have 2 hamburgers. She has found that 1 hamburger takes about one-quarter pound of meat. How much hamburger meat should she buy?

a. 80 lb b. 20 lb c. 200 lb

5. A border for a flower garden is 5 m long. Marigolds are planted on the border 25 cm apart. About how many marigolds make up the border?

a. 20 b. 50 c. 200

6. Mr. Tubbs took a trip by car and filled his tank 4 times. The fill-ups took 14.3 gallons, 15.1 gallons, 13.9 gallons, and 15.8 gallons of gas. The average price per gallon was 115.9¢. About how much did Mr. Tubbs spend for gas?

a. $7.00 b. $70.00 c. $700.00

7. The town of Sun Center is only about 30 miles south of Wet Rock but averages 154 more hours of sunshine per year than Sun Center. About how much more sunshine per day does Sun Center get than Wet Rock?

a. 1.5 h b. 2.5 h c. 30 min

8. Tess brushes her teeth twice a day. She has a tube of toothpaste that contains 150 mL. If Tess uses about 1.5 mL each time she brushes, about how many weeks will her tube of toothpaste last?

a. 7 weeks b. 50 weeks c. 100 weeks

Estimation helps you to decide if an answer obtained is reasonable.

Problem:
A swimming pool has a capacity of 48,000 gallons. If the pool is filled at the rate of 25 gallons a minute, how long will it take to fill the pool?

Solution:
Estimate the answer.
48,000 gallons rounds to 50,000 gallons.
50,000 ÷ 25 = 2000 min to fill the pool.
2000 ÷ 60 is about 33 h to fill the pool.

Calculate the answer.
48,000 ÷ 25 = 1920 min to fill the pool.
1920 ÷ 60 = 32 h to fill the pool.

The estimated answer of about 33 h tells us that the calculated answer is probably correct.

Estimate a reasonable answer. Then calculate the exact answer.

1. Last year Shannon earned $546.00 by babysitting. What were her average weekly earnings from babysitting?

2. One day, the donut shop sold 108 dozen donuts at the rate of $3.75 per dozen. How much money was taken in for the donuts?

3. At Cedarbush School, there are 784 students. About one fourth of the students watch TV more than 3 hours a day. About how many students is that?

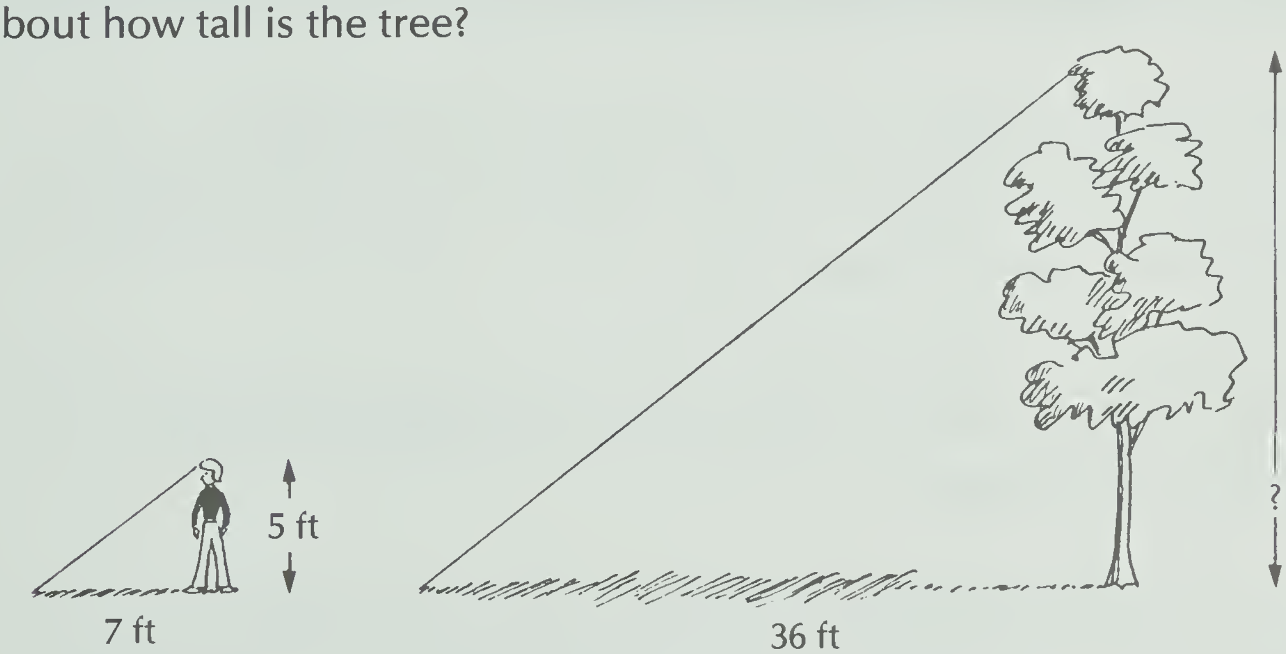
4. On a fishing trip, Josh caught a 15 pound and a 6 pound salmon. At the time, the supermarket price for salmon was $12.50 per pound. What was the supermarket value of the salmon caught by Josh?

5. A city census was taken and a count of 119,800 city residents was found. If 49% of the residents are males, how many males are in the city?

6. On a golf course, 5000 square yards of turf are to be treated with a special weed and feed mixture. A 25 pound bag of the mixture will treat about 500 square yards of turf. About how many bags of the mixture will it take to treat the turf?

7. Helga drinks 1 glass of milk daily.
A glass holds about 300 mL.
How many liters of milk does she drink in 1 year?

8. Marg is 5 feet tall. At a certain time of day, her shadow length was 7 feet long. At the same time, a tree cast a shadow 36 feet long. About how tall is the tree?



9. At a showing of *Temple of Doom,* there were 960 people. Prices for adults were $4.50; for youths, $3.50; and for senior citizens, $2.50. If there was an equal number of adults, youths, and senior citizens at the movie, how much money was taken in from ticket sales?

UNIT 13 Using Other Resources

Atlases, almanacs, newspapers, or encyclopedias contain the facts needed to solve some problems.

Problem:
Which is greater, the combined population of Canada's two largest provinces, or the population of all the rest of Canada?

Solution:
Investigate and compare the populations:

Quebec	6,438,403	All of Canada	24,343,181
Ontario	+ 8,625,107	2 largest provinces	− 15,063,510
	15,063,510		9,279,671

$$15{,}063{,}510 > 9{,}279{,}671$$

The two most populated provinces have more people than all the other provinces and territories combined.

Use other resources to find the facts you need to solve the problem.

1. How long would it take a trout to swim the entire length of the Yukon River if it could swim 2 miles per hour?

2. How much farther is it from Kansas City to Minneapolis than from Minneapolis to Chicago?

3. About how many cats would weigh the same as an elephant?

4. What animal is the largest in the world? What is its weight? What is its main food?

5. How much greater was the winning high jump distance in the 1984 Olympics than it was 52 years earlier? What were the high jumpers' names who won the gold medals for these events? What countries were they from?

6. How many miles is it to drive from your town to the most distant point (that can be reached by road) in your state?

7. What was the population of the United States in 1980? Compare this to the population in 1970 and 1960.

8. About how many telephone numbers are there in your local telephone book? How does the number of telephones compare with the population of the telephone district?

9. Mercury is a very heavy liquid metal. How much more is the weight of 16 fluid ounces of mercury than the same amount of water?

10. How many years after Arizona (our 48th state) was the next state admitted to the Union?

11. How many miles of seacoast does the United States have? How does this compare to Canada?

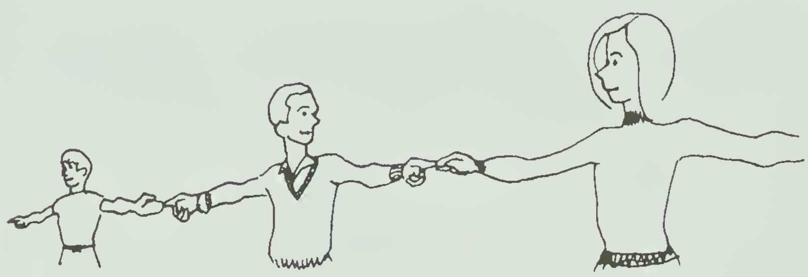
12. What is the total driving distance from San Francisco to Reno, Salt Lake, Denver, and Kansas City?

A survey or other activity can provide the information necessary to solve a problem.

Problem:
If the entire class lined up with arms extended and finger tips touching, how long would the line be?

Solution:

One way



Form the line described.
Measure the length.

Another way

Select an average-sized student. Measure his or her arm span. Multiply the arm span by the number of students in the class.

1. About how many students with arms extended and touching fingertips would it take to form a continuous line from Memphis to Louisville?

2. About how many books are in the school library?
About how many of these books are on the reference shelves?

3. About how many words are there in one of your textbooks?

4. If basketballs could be piled one on top of the other, how many basketballs would it take to reach to the top of a basketball net?

5. Measure a large U.S. commemorative postage stamp and also a smaller regulation stamp. If the bulletin board were covered completely with the smaller stamps, how many more of those stamps would be used than if the large commemorative stamps completely covered the bulletin board?

6. Find out how long your pace is. How many paces would it take to cover the distance from one end of a school hallway to the other?

7. How tall is the school flagpole on the school grounds?

8. About what fraction of the number of pages in your dictionary have words beginning with the letter "S"?

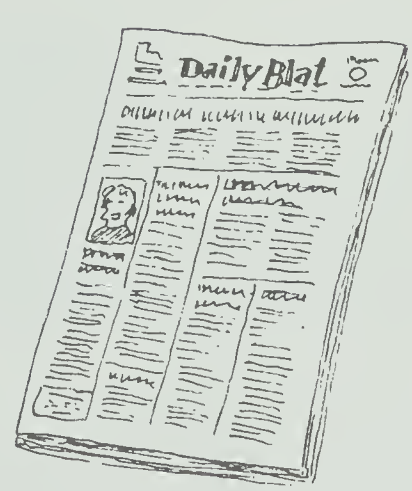
9. Make surveys to find out the following:

   a. Most popular television program
   b. Favorite spectator sport
   c. Favorite rock star
   d. Favorite dessert
   e. Most disliked food

   Record the results of your survey. Display the results on a graph or chart.

10. How many pounds of hot dogs would it take to supply lunch for every person in the school?

11. About how many words appear on the front page of your daily newspaper?



UNIT 14 Review Problems

1. If the 52 cards in a deck are arranged in a line with each card touching the edge of the next card, how long will the line be?

2. Della worked the first 12 problems of a 40-problem test in 20 minutes. The time allowance for the test was one hour. If Della continued the test at the same rate, how many problems will she have worked when time is up?

3. The ad shown below appeared in a newspaper. Mr. Becker has two cars and he had engine tune-ups done on both of them. His bill for the two tune-ups was *about* $112.00. What is the number of cylinders on each of Mr. Becker's two cars?

12-MONTH TUNE-UP

46.95 4 cyl.

56.95 6 cyl.

64.95 8 cyl.

Includes 12-Month Service Warranty.

4. Below are the parking rates for 2 garages:

U-PARK	$0.50 for the first hour $0.30 for each additional hour
CITY PARK	$1.00 for the first hour $0.20 for each additional hour

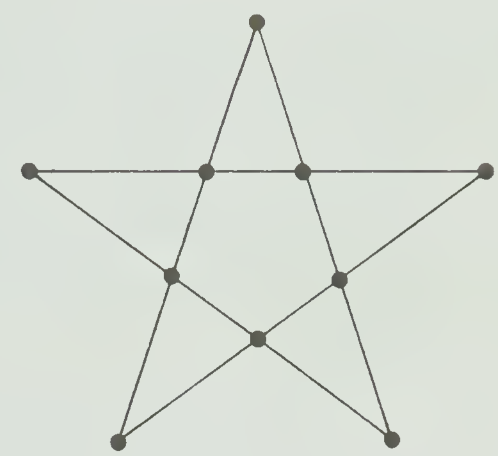
For what number of hours will the parking costs be the same for the two garages?

5. The table gives the approximate distances to the sun in kilometers from several planets. If Mercury is represented on a bar graph by a bar 4 cm long, how long a bar should represent the Earth?

Mercury	60,000,000
Venus	110,000,000
Earth	150,000,000
Mars	220,000,000

6. A palindrome is a number which is read from right to left exactly as it is from left to right. Examples are 575, 8338, 35,253, and 7777. How many numbers between 11 and 999 are palindromes?

7. It is possible to have 10 plants arranged so that there are 5 rows with 4 plants in each row. This can be done by arranging the plants in the pattern of a 5-pointed star as shown. Draw a figure which shows how you could have 12 plants with 6 rows and 4 plants in each row.

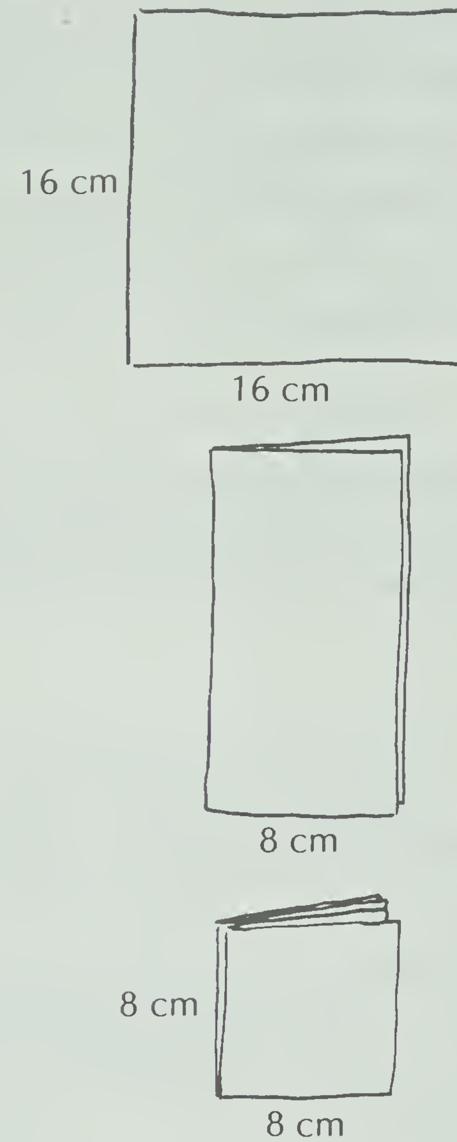


8. At a track meet, Mountainview School scored twice as many points as Clearbrook School. Shadylane School scored three times as many points as Mountainview. Altogether there were 198 points scored in the track meet. What were the point totals for each school?

9. There are 2 routes from the base of Mt. Cupcake to the top. One route is a steep climb of 1800 m. The other route is a hiking distance of 6600 m. Luke estimates that he can climb the steep ascent at the rate of 500 m/h. Melanie estimates that she can walk the hiking route at the rate of 2 km/h. If their estimates are accurate, who would reach the mountain top first, and how much more quickly?

10. Start with a paper square which is 16 cm on each side. Find its area. Fold the square in half and find the area of the rectangle which is formed.

Next, fold that rectangle in half and find the area of the square which is formed. Continue folding into rectangles or squares six times. What is the shape of the figure formed after the sixth folding?
What is the area of that sixth figure?



11. Find the perimeter of each of the above squares and rectangles before it is folded into a smaller shape. Make a list of the perimeters which are formed. What is the perimeter of the figure after the sixth folding? the eighth folding?

12. Hank does engine tune-ups in his father's garage. In one week, he replaced all the spark plugs on 17 cars. The cars had either 4 or 6 spark plugs each. The total number of spark plugs replaced by Hank was 90. How many cars required 4 spark plugs each?

13. How much does a family save if it buys milk in 3 half-gallon cartons instead of 2 quart cartons and uses 24 quarts of milk per week?

14. The longest postage stamp ever issued is the 1913 Express Delivery stamp of China which measures 24.8 cm long and 7.0 cm wide. The smallest stamp ever issued is the 1 peso of the Columbian state of Bolivar issued in 1863-1866. This stamp is 0.94 cm long and 0.78 cm wide. About how many times would the length of the smallest stamp fit into the length of the largest stamp?

15. A popular magazine sells on the newsstand for $2.00. To attract mail subscriptions, they offer these choices:

(The longer your subscription, the more you save!)

☐ 9 months, $12 ☐ 12 months, $16 ☐ 18 months, $23 ☐ 24 months, $28

Name

Address

City State Zip Code

☐ This is a new subscription. ☐ This is a renewal.

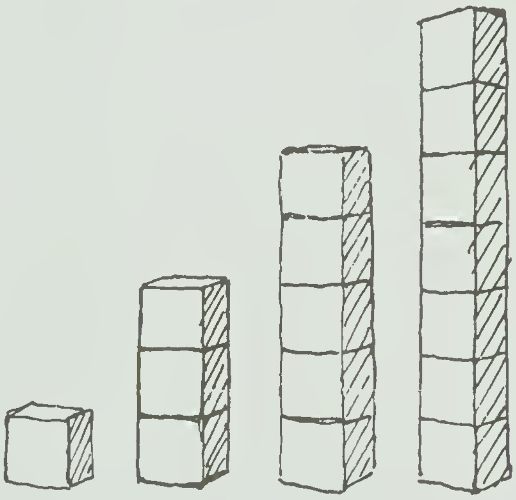
☐ $__________ enclosed. ☐ Bill me later.

Save up to $35. off the cover price!

What is the cost per magazine for each subscription time period? Can you save $35 off the cover price?

16. Mr. Kimble, a grocer, checked his supply of milk and counted 85 containers. Some were 1 qt cartons and others were half-gallon cartons. Altogether there was a total of 30 gallons of milk. How many of the containers were half-gallons?

17. Stack blocks with 1 block in the first column, 3 blocks in the second column, 5 blocks in the third column, and so on for 8 columns. What is the fewest number of blocks that you could move to make all 8 columns have equal height?



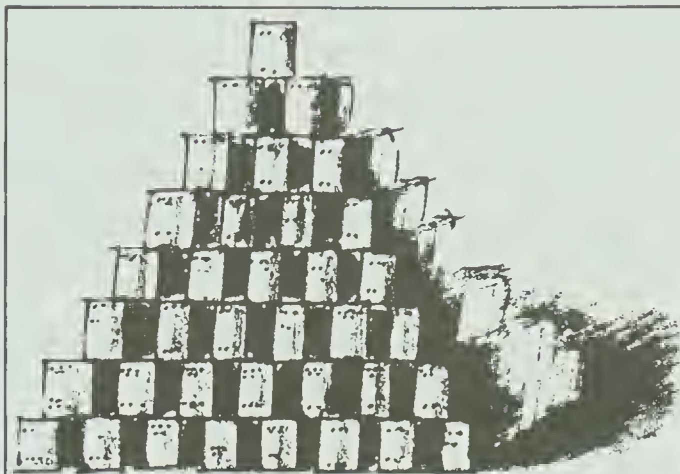
18. Todd and Rod decided to bike to the city park. Todd took the long way. He went north for 4 blocks, east for 6 blocks, south for 2 blocks, and arrived at the park. Rod took the short way. He went east for 4 blocks, north for 2 blocks, east for a certain distance, and then reached the park. How much farther did Todd have to bike than Rod?

Teacher's Resources

Problem Solving

What is it?

Definitions of mathematical problem solving and descriptions of its scope continue to be developed by teachers and researchers. George Polya has said that to solve a problem is to find a way where no way is known offhand, to find a way out of difficulty, to find a way around an obstacle, to attain a desired end that is not immediately attainable by appropriate means.



A supermarket manager wants to build a square pyramid with cans of apple juice. The bottom layer is to be 8 cans wide.
How many cans will the display use?

Its Importance

Achieving success in problem solving continues to be an important student objective for teachers at all grade levels. Recent mathematics assessments have highlighted the fact that while most students do well on computational items, they perform poorly on exercises requiring problem solving or application of mathematical skills.

The National Council of Teachers of Mathematics Priorities in School Mathematics Project (PRISM) found that classroom teachers, mathematics educators, and lay people all supported the position that problem solving should receive greater emphasis in the curriculum. In their recommendation for school mathematics in the 1980's, the N.C.T.M. proposed that problem solving be the focus of school mathematics.

As a result of these concerns, mathematics educators across the United States have made problem solving a major focal point.

Where do we start?

By its very nature, problem solving is frustrating. Since answers do not come quickly, it is easy to become impatient and avoid future problem situations. Consequently, it is crucial that a positive, low pressure, relaxed classroom atmosphere be used so that student interest in and success with problem solving can be attained.

Teaching Suggestions

1. Allow time for group discussion. Talk about the facts presented. See that the students *understand* the problem. Share student ideas and *strategies* for solving the problem.

2. Encourage students to work in pairs. Motivation is greatly increased when students can share ideas. Students can learn from each other.

3. Do not assign work as you would from the basic textbook. *Challenge* the students to do a certain number of problems and then allow them to do a few problems of their own choice. Stress quality rather than quantity.

4. Encourage flexibility. Accept different problem-solving strategies. Share different solutions.

5. Change standards for the appearance of student work. The student work will not usually look like a textbook assignment. Do not always require number sentences to be written for the answer. Encourage pictures and diagrams. Students should be able to write *concluding statements* to the problems.

6. Have the students keep a problem-solving notebook. Similar problems which have previously been solved can then be referred to.

7. Provide appropriate *hints* without giving away the answer.Remind the students of the various problem-solving strategies. Pictures or diagrams can often be a hint to a problem's solution.

8. Simplify problems, as needed. Sometimes a problem is overwhelmingly complex and it is better to start with a simpler problem.

9. Extend problems, creating variations to similar, previously-solved problems. For example, once the following problem is solved, try the given variations.

> If 5 children trade one baseball card with each other child, how many different trades would take place?

> *VARIATIONS*
>
> (1) Change the number of children.
> (2) Change the number of cards traded.
> (3) Change the *given* and *wanted* information. (Give the number of trades. Ask for the number of children.)

10. Create a problem-solving center, puzzle of the day, and bulletin board. Maintain a learning environment rich in manipulatives and ancillary supplies ready for use in problem solving.

Unit 1: Organizing Information

Objectives

List information in such a manner that problems are easier to understand and solve. Extend the skill of systematic listing to making tables.

Notes

Many students have great difficulty knowing where to start in a problem. By simply listing information, a process that is easier than trying to comprehend the whole problem at once, students have the opportunity to arrive gradually at an understanding of what is given and what is sought.

The entire class should discuss the example on page 2. It is quite easy to understand, involves relatively simple numbers, and invites a number of different possibilities for which even slower students can contribute examples.

Encourage the students to make their lists systematically. This involves starting with an extreme example at the top (highest or lowest set of numbers in the sequence).

In problem 1, page 2, students should list first Freda's age at birth and mother's age at birth. The remaining sets of numbers in the list should follow in natural order until the students come to the correct answer of 11 years for Freda.

In a problem like number 3, you will likely have to point out that as students make their lists, some number combinations are immediately rejected. Observe the following partial list:

Number of 10¢ Stamps	Number of 15¢ Stamps	Value
10	0	\$1.00 OK
9	1	\$1.05 Reject

Pages 4 and 5 extend the skill of systematic listing to an even more systematic list, that of making a table. It's a good idea to discuss the example with the class, probably building up the table using an overhead projector. Allow the students time to think and reason what the next entry in the table ought to be. Some may want to race ahead to obtain an answer. You should emphasize the importance of completeness and orderliness. Keep in mind that this completeness and orderliness is absolutely essential because students will be solving problems in Unit 3 by looking for patterns. Without a very orderly and complete table, it will be difficult if not impossible to note patterns later.

Problem 7, page 5, is difficult but it has some motivational advantages. Discuss first what students know about falling bodies. Intuitively they probably know that objects fall faster (accelerate) when dropped. Students must assimilate two ideas here: **1.** That an object falls a certain distance in each *interval of one second*; **2.** An object falls a *total distance* after any number of seconds. The table entries corresponding to total distance are obtained from the *previous total distance* and the *distance the ball fell in the present second*.

Unit 2: Guessing and Testing

Objective

Solve problems by systematically improving guesses regardless of the extremity of an initial guess.

Notes

Instead of using the guesses shown in the example problem on page 6, allow the students to come up with any other initial guess. It's very important that they see that, even if their first guess is far off, by intelligently considering the direction to be taken, *any* initial guess will help students to reach the correct answer. No problem-solving strategy invites participation more easily than Guessing and Testing. Every first guess is considered a "correct" answer. Therefore, students must think and consider before the next guesses.

Another advantage of this method is that students can quickly determine whether or not the guess has arrived at the correct answer.

There is a very close relationship between Making a List and Guessing and Testing. In making a list there will be complete sets of numbers starting with one extreme and going to another. In Guessing and Testing, students skip over some sets of numbers especially if the initial guess is very far off the goal. Also, in making a new guess, a student may leap past the goal and have to reverse direction. Just about any problem which can be solved by Guessing and Testing could also be solved by Making a List. Generally, a Guess-and-Test strategy is used when there is only one correct answer. In a Make-a-List strategy, there are frequently many correct answers or listings all contributing to the final goal.

In problem 9, page 7, a slight complication may result because there are three types of stamps among the 15 stamps. You may choose to help students understand the importance of the fact that there is the *same* number of 40¢ and 60¢ stamps. If this is not well understood, the numerous possibilities for obtaining three whole numbers which add up to 15 will be very discouraging to the problem solver.

Unit 3: Looking For Patterns

Objective

Solve problems by observing patterns in a list or table.

Notes

The success of this unit may depend much upon how well students practiced skills in making and using tables in Unit 1. One might have lists without patterns, but it's hard to visualize any number patterns without having organized lists or tables first.

As with all units, a general class discussion of the example problem on page 10 is essential. Discuss several pattern problems with the entire class. Here are excellent opportunities to elicit suggestions from almost any student including slower students.

The example problem on page 12 needs clarification because of possible different interpretations. In the example, it doesn't matter if vanilla is on top and chocolate on the bottom or vice-versa. Some students may argue with that. If they object strongly, there's nothing wrong with agreeing to an interpretation with which most people are happy. In the example, capital letters are used to identify the various flavours. If this is too abstract for your students, allow them to write out the full words. Letters do cut down the pencil work and make lists easier and pattern observation more apparent.

In problem 1 on page 12, some students may try to answer the question by drawing a set of 6 lines each of which intersects the other five. There are some visual difficulties here even if the lines are properly drawn. This is a good way to illustrate how one's recognition of a pattern may be more useful than brute force even meticulously applied.

In problems 5 and 6, page 13, an overhead projector with simple but carefully drawn diagrams will help considerably. Do not proceed until students can actually see the various rectangles.

Unit 4: Too Many and Too Few Facts

Objective

Develop skills in identifying essential facts and in determining necessary missing information.

Notes

Students may be quite uncomfortable in dealing with problems of these types. Their experience generally is that there is almost exactly enough and just enough information to solve problems. Of course this is not the way it is in real-life activities, so these problems are actually more realistic. The problems in this unit appear more like traditional, textbook problems aside from the amount of information given.

Discuss the example problems on page 14 and 16 before the students solve the problems of this unit.

Unit 5: Using Diagrams

Objectives

Develop skill in making use of a pre-drawn diagram to solve problems. Draw diagrams that will help solve problems.

Notes

The example on page 18 of the fence-post problem is useful because it illustrates clearly that intuition may be faulty in some problems. The corner posts which are on two sides of the garden are easily overlooked under the simplistic perception of four sides, four posts to a side, $4 \times 4 = 16$. This provides a good motivator for drawing a diagram.

In the first section, diagrams are provided because it is easier to use a diagram than it is to construct one. Construction of the diagrams on pages 20 and 21 may be very difficult for many students. Several of these problems may require teacher assistance or class discussion expedited by use of the overhead projector. It is a good idea to allow students to make diagrams for display to the class even if they are faulty. Let the students observe any imperfections or omissions in the diagrams and correct each other.

Problem 4, page 19, is recommended because students must finish the remaining diagrams that show how four squares can be placed together. Then they must examine each one to determine if there is one which has the smallest perimeter as asked in problem 5.

If students have great difficulty constructing the appropriate diagrams on pages 20 and 21, you may provide help to show them a correct diagram and then ask them to complete the solution using the diagram.

Unit 6: Using Equations

Objectives

Develop skill in writing equations. Use equations to guess and test possible solutions to a problem.

Notes

The problems in this unit are in general easier Guess-and-Test problems than those seen in Unit 3. Here the emphasis is on the ability to *use* an equation to substitute guessed values of an unknown and to test to see if they "work."

For the example problem on page 22, the student is to "plug in" a guessed number for *B*, multiply by 6, and then check to see if the resulting number is 90. This process stops when the guessed number does indeed produce a 90 after multiplication by 6.

At this stage, students should have an intuitive understanding of addition, subtraction, multiplication, and division situations which will enable them to construct a proper equation.

The problems in the first section of the unit can be solved with only one equation. On pages 24 and 25, two equations are needed. The problems are constructed so as to suggest two equations naturally, and the conditions for both of the equations must be satisfied.

Of course these guess-and-test problems can be solved without direct construction and use of equations. However, even if such equations are *not* actually constructed and written down, the student will be using them implicitly in their guess-and-check methods. Consider the thinking process for problem number 4 on page 25. A dress costs $6.00 more than a hat. Together, the cost is $40.00. Suppose the hat costs $1.00 and the dress $7.00. The total cost is $8.00. The answer is not correct, but it's a start and will lead to better guesses. In any case, the student has made use of two equations at least in his head. Of course we should try to encourage him/her to write the equations down, unless he/she can't cope with this rudimentary algebra at this time.

Unit 7: Review Problems

Objective

Review the following problem-solving strategies:

1. Organizing Information
2. Guessing and Testing
3. Looking for Patterns
4. Too Many and Too Few Facts
5. Using Diagrams
6. Using Equations

Notes

Students use the strategies practiced in the first six units to solve the problems of this review unit. It is important to allow students flexibility in solving problems. They may choose to employ strategies which are not the simplest or most effective. However, any strategy that works is acceptable. It can be very edifying for students to see problems solved in more than one way, especially when members of the class have achieved those different methods to solution.

Problems 19 and 20 on page 29 go together. Many students will be very much surprised that rectangles with the same perimeter have different areas. It may be wise to provide a start on problem 19, perhaps suggesting a rectangle which is 11 units long and 1 unit wide, and having students proceed from there. With the systematic list that results, they will also see that a rectangle which is 11 by 1 is the same as a rectangle which is 1 by 11.

Problem 22 is quite a challenging one. First of all, the problem itself may not be clear. As an extension of problem 21, the idea of one side of the rectangle essentially falling out of the picture may be hard to comprehend. After solving problem 21, it is natural to make the three sides of the rectangle in problem 22 equal. Some students may hesitate to try other dimensions because of their reasoning on the basis of the result in problem 21. This particular problem might best be solved as an entire class, allowing students to take as much initiative as possible.

Unit 8: Organized Lists

Objective

Develop skill in using an organized list for solving problems.

Notes

In the first half of this unit, data from which problems are posed are presented to the student. The data are presented in an organized or semi-organized form. For the latter, students are expected to assemble the data into a more presentable and workable form. Some problems are easier to solve when thought is given to how the data should be arranged.

In the problems where tables are already presented, students still need to take time to understand what is in the table. Most people underestimate the vast amount of data which a single table may contain. Problem 3 on page 31 contains a great deal of data which will appear complex and confusing to students at first. This table, like many

others, must be examined in parts at first. Furthermore, time must be taken so that students can understand the Earth's distance to the Sun can be taken as one unit instead of about 149,000,000 km. Ask questions that lead students to better understanding of this table. For example, "How many times as far from the Sun is Saturn than Earth?" Students can also be encouraged to make up their own problems using the table.

Point out the value of making lists as the solution to the example problem on page 32 is discussed. The remaining part of this unit also requires lists to be made, but the data are not so explicitly presented.

Unit 9: Ratio, Proportion, and Percent

Objective

Use ratio, proportion, and percent to solve problems.

Notes

In the first half of this unit, the concepts of ratio and proportion are practiced in combination with use of a rate table. The example problem on page 34 provides two methods of solving the problems. In the table, it is shown that the same rate results whether we add the same number to the numbers in the preceding first and second rows of the table or whether we multiply both numbers by the same factor. When a problem requires going from a ratio in simplest form to one which uses large numbers, the technique of doubling and multiplying by 10 is very useful in getting to the "large numbers" more quickly. The tables used here therefore differ in that respect from tables which were used in earlier units, especially Unit 3, Patterns. Looking for a pattern usually requires an increment of one without skipping number pairs such as results when one doubles or multiplies by 10 the preceding table entries.

Also as shown in the example, a second method for solving ratio and proportion problems is also given in the example. This is the method which is presented in the text. Students should practice both methods for a fuller understanding of the concepts and for increasing success in problem solving.

In problem 5, the table entries should reflect the *decreasing* entries in the table. Here one must proceed from large numbers that form a ratio to the simplest ratio, unlike the preceding problems.

Problem 8 is more challenging but may motivate some students who are interested in baseball. The crucial step in this problem is to see that solution depends on first finding how many games are equivalent to 225 innings.

For the percent problems on pages 36 and 37, the table method used in the preceding two pages is carried on. The only difference is that the tables in this part will always have as a goal one entry which is the number 100. The proportion method is also shown. This method can be useful to reinforce the concept of percent as a relationship between a number and 100.

Unit 10: Multi-Step Problems

Objectives

Solve problems which require solution of at least one more smaller problem.

Notes

The example problems on pages 38 and 40 typify the procedures for solving the problems in this unit. Because there is always more than one step, care must be taken to ensure that students understand what the goal of the problem is. It is possible that part of the problem could be solved by a student with the belief that all of the problem was solved. As there are considerable data in each of these problems, it might be advisable to discuss some of the problems with the entire class with the emphasis on assimilating all of the facts and identifying the main goal in the problem.

There is a wide variety of contexts for these problems. Perhaps it might be most beneficial to allow students to select problems according to their interests such as hiking, sports, airplanes, etc. For example, problem 1 on page 40 may be of interest to a few students who follow professional soccer but of very limited interest for others.

Unit 11: Making a Simpler Problem

Objectives

Solve complex problems by rounding off numbers and/or discarding irrelevant information. Solve a sub-problem in order to solve the major problem.

Notes

By this time, students have had enough practice with decimals to begin dealing with problems that contain rather awkward-looking decimals. Appearances can be deceiving and devastating to a student's motivation to consider a problem seriously. It may be helpful to have students rewrite problems using much simpler numbers (rounding off those in the problem) and by including only the data which are necessary to solve the problem.

In the case of problems which present facts in table form, it is sufficient for students to identify the facts that are needed. For example, in problem 2 on page 43, there are three lines of data in the table, but only the facts dealing with the 200 m and 1500 m run are necessary to solve the problem. At the same time, the presence of additional information in the table is more realistic as this is how we are confronted with information in general. The other data also provides a basis for additional problems that might be posed especially by students who happen to be motivated with the context of the problem. A student could easily ask a question similar to the problem, e.g., "What is the speed of each runner in meters per minute?"

The sub-problems on pages 44 and 45 represent a different aspect of making or solving a simpler problem. As the example indicates, before a student can count all the squares, he has to identify the existence of some larger squares within the figure, and to be able to count those other squares as well as the more obvious ones.

Another way to make a simpler problem is to take an extreme case, solve it, and then try to solve the original problem. For the example problem, this would mean looking at one step, two steps, three steps, and then four steps and five steps. If all of these are done, it then becomes possible to observe a pattern from which the solution to staircase problems of this type all become solvable.

In problem number 4 on page 45, there are two distinctly identifiable problems. One is a sub-problem, that of finding the width of the picture frame, the other is the main problem of finding the area of the frame. Yet, although the word, sub-problem, may suggest an easier problem, here the solution of the sub-problem is actually more difficult than solving the main problem.

Problem 6 on page 45 has several sub-problems, each reasonably simple.

Unit 12: Reasonable Answers

Objective

Develop an awareness of reasonableness in answers.

Notes

In the first two pages of this unit, students are presented with a choice of answers, most of them differing by factors of 10. The object here is to have students consider numerals that look alike except in the placement of the decimal or the number of zeros. Too often students give little thought to the actual size of the number as related to the data in the problem. As a result, whatever figure has been calculated is accepted as correct. The frequency of decimal point errors and improper number of zeros in answers makes it essential to have students take time and to practice with the reasonableness of answers.

During the discussion of the example problem on page 46, stress that, of the 3 possible answers given, only one is reasonable. Even the disinterested basketball observer should recognize that 5 points or 500 points as a game average does not make sense. Other problems on the first two pages of this unit may require some initial rounding off and estimation of answers before a decision can be made about answer reasonableness. For example, in problem 6 on page 47, rounding the number of gallons of gas used should give a figure of about 60, and rounding the price to about $1.16 should help the student to eliminate the erroneous answers of $7.00 and $700.00.

The example problem on page 48 is more difficult than most of the problems that follow and is very worthwhile to discuss carefully with the class. Insist that students estimate the problems on pages 48 and 49 first so they then can check the reasonableness of their calculated answers.

Unit 13: Using Other Resources

Objective

Use data collected from outside resources to solve problems.

Notes

Students will realize that often not all data are immediately available for the solution of a problem, and that such data can be obtained by investigating printed resources like almanacs, fact books, encyclopedias, etc. or by actually performing a count, or a survey or doing an experiment.

This unit can actually improve the motivation of students instead of frustrating them when information is lacking. The act of going to another resource whether in the classroom or the library is a departure from the desk and textbook method that may become tiresome. Furthermore, the act of locating information is a stimulating experience, once it has been sought for, and students may pick up other interesting information incidentally. This may lead to formulation of additional problems.

These kinds of problems could be used as part of a individual or group project. Some problems could be integrated with other subjects such as social studies, or physical education.

While the first two pages focus upon information to be obtained from printed resources, the second two pages consist of problems which can be solved by making measurements, or by experimentation, or other physical rather than sedentary activities.

In problem number 4 on page 52, it would be difficult to actually pile basketballs atop one another, but students could solve the problem by measurement of the diameter of the basketball and the height of the basketball net. Problem number 7, page 53, is certainly more interesting than solving similar problems where the measurements have already been given.

Problem number 9, page 53, suggests various surveys which could be made by students in the school or the community. Any such survey would have to be supplemented by some instruction on what would constitute a random and unbiased sample. This activity lends itself very well to construction of attractive displays such as a graph or chart.

Unit 14: Review Problems

Objectives

Review the following problem-solving strategies:

1. Organizing Information
2. Guessing and Testing
3. Looking for Patterns
4. Too Many and Too Few Facts
5. Using Diagrams
6. Using Equations
8. Organized Lists
9. Ratio, Proportion, Percent
10. Multi-Step Problems
11. Making a Simpler Problem
12. Reasonable Answers
13. Using Other Resources

Notes

More than one strategy may be used to solve any one of the problems in this section.

Answers

ANSWERS

UNIT 1

Page 2 **1.** 11 years **2.** BBBB, BBBG, BBGG, BGGG, GGGG **3.** two 15¢ stamps and seven 10¢ stamps; four 15¢ stamps and four 10¢ stamps; six 15¢ stamps and one 10¢ stamp; ten 10¢ stamps

Page 3 **4.** four at 60¢ and six at 30¢ **5.** 1234, 1243, 1324, 1342, 1423, 1432, 2134, 2143, 3124, 3142, 4123, 4132, 2314, 3214, 2413, 4213, 3412, 4312, 2341, 3241, 2431, 4231, 3421, 4321 **6.** 5 spiders and 7 beetles **7.** 3 sonnets and 14 limericks **8.** 8, 8, 9, 9, 7 **9.** 9 quarters

Page 4 **1.** 11 d **2.** 19 triangles **3.** 64 great, great, great, great grandparents

Page 5 **4.** 55 blocks **5.** $2.75 **6.** 1024 cells **7.** 10 s **8.** 2:00 P.M.

UNIT 2

Page 6 **1.** 8 cans **2.** 10 sesame snaps **3.** $16.00

Page 7 **4.** 5 times **5.** 4 apples, 3 bananas, and 1 peach **6.** $1800.00 **7.** 62 dimes and 38 nickels **8.** 40 children **9.** three 30¢ stamps **10.** 2 Giant Burgers

Page 8 **1.** seventy half-gallons **2.** 126 girls **3.** 146 votes
4.

4	9	2
3	5	7
8	1	6

Page 9 **5.** 14 cars **6.** 146 **7.** 14 years **8.** 16 planes **9.** 68 **10.** 66 and 34 **11.** 1, 2, and 3 **12.** 24

UNIT 3

Page 10 **1.** 2, 4, 8, 6, 2, 4, 8, 6, . . . **2.** 6, 6, 6, . . . **3a.** 3, 9, 7, 1, 3, 9, 7, 1, . . .; **3b.** 7, 9, 3, 1. 7, 9, 3, 1, . . .; **3c.** 9, 1, 9, 1, . . .

Page 11 **4.** 6, 2, 8, 4, 0, 6, 2, 8, 4, 0, 6, . . . **5.** 5445, 6534, 7623, 8712, 9801 **6.** $345 \times 1001 = 345{,}345$, $627 \times 1001 = 627{,}627$ **7.** 100 dots **8.** 9, 20 **9.** 17 numbers, 39 numbers **10.** 81, 400

Page 12 **1.** 15 points **2.** 37 sides

Page 13 **3.** 12; 30 **4.** 22 pieces **5.** 36 rectangles **6.** 36 rectangles

UNIT 4

Page 14 **1.** 75 g less than 1 kg **2.** 2.5 h **3.** 41 to 38.

Page 15 **4.** about 4 times **5.** 24 000 km^2 **6.** Lake Erie and Lake Ontario **7.** $2.68 **8.** $8.86 **9.** $2.15

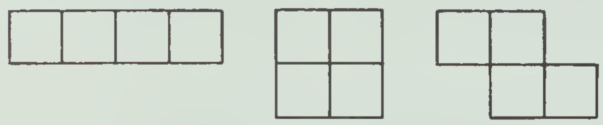
Page 16 **1.** missing fact: the number of general admission fans **2.** 1090 mi **3.** missing fact: length of Nile River **4.** missing fact: length of Mississippi River

Page 17 **5.** 1 h **6.** missing fact: depth of Lake Michigan **7.** missing fact: the subscription rate Mr. Perkins chose **8.** missing fact: lengths of 2 sides **9.** missing fact: how many minutes it takes Chris to have a shower **10.** $598.50

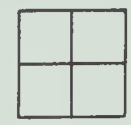
UNIT 5

Page 18 **1.** 48 rose bushes **2.** 13 valves **3.** 1440 cm

Page 19 **4.**



5. 20 cm



6. 9 m² **7.** 1019 cm **8.** Ali to Eli to Fali to Deli to Cali to Bali to Ali, 330 km

Page 20 **1.** 20 km **2.** 1100 sq ft **3.** 243 ducks

Page 21 **4.** 24 routes **5.** 210 cans **6.** Ian **7.** 6 students **8.** 2 cars **9.** Badgers

UNIT 6

Page 22 **1.** $3 \times F = 51$, $17.00 **2.** $34 + B = 200$, 166 cards **3.** $52 + M = 99$, 47 kg **4.** $18 \times L = 360$, 20 m^2

Page 23 **5.** $\$4.78 + M = \$10.00$, $5.22 **6.** $83 + T = 108$, 25 games **7.** $48 + D = 263$, 215 dots **8.** $137 + M = 1{,}000{,}000$, 999,863 dishes **9.** $9 \times W = 300$, about 33 lines **10.** $25 \times S = \$8.75$, $0.35 **11.** $17 + M = 22$, 5 min **12.** $5 \times T = \$42.50$, $8.50 **13.** $53 + W = 120$, 67 games

Page 24 **1.** $A + B = 15$, $A \times B = 56$, 7 and 8
2. $C + E = \$2.00$, $C - E = \$1.60$, $1.80
3. $T + F = 46$, $T - F = 8$, Toads 27, Frogs 19

Page 25 **4.** $D + H = \$40.00$, $D - H = \$6.00$, $17.00
5. $F + P = 352$, $F - P = 18$, father is 185 cm, Paul is 167 cm
6. $F + M = 8400$, $F - M = 600$, 4500 females
7. $T + E = 205$, $T - E = 33$, Timmy is 119 lb, Eddie is 86 lb
8. $A + B = 68$, $A - B = 6$, front 9: 37 strokes; back 9: 31 strokes
9. $2A + 2B = 26$, $A \times B = 40$, length: 5 cm, width: 8 cm
10. $P + G = 20$, $4P + 2G = 70$, 15 pigs and 5 geese
11. $A + B = 52{,}000$, $A - B = 12{,}000$, old car: 32,000; new car: 20,000
12. $A + T = 65$, $A - T = 9$, Anna: 37 guppies; Tom: 28 guppies

UNIT 7

Page 26 **1.** 24 outfits **2.** 831, 832, 833, 834, and 835 **3.** 378 students **4.** $19.50 **5.** 10 dinners **6.** $45.00

Page 27 **7.** 260 km **8.** 5120 people **9.** 12:13 **10.** missing fact: hours worked in one week **11.** $16,402,000 **12.** 10 years **13.** missing fact

Page 28 **14.** $7.42 **15.** $2,300,000 **16.** 7 coins: 1 dime, 1 nickel, and 5 pennies **17.** 12 in. long, 6 in. wide **18.** $10 for soccer ball, $4 for a baseball

Page 29 **19.** 6 rectangles **20.** the 6 by 6 rectangle **21.** 40 pages **22.** 50 ft by 25 ft **23.** 2200 people **24.** 91, 92, and 93 **25.** The sum of the outer digits equals the sum of the center digit.

UNIT 8

Page 30 **1a.** false **1b.** true **1c.** true

Page 31 **2a.** less **2b.** rattlesnake **2c.** no **2d.** 3 **3a.** true **3b.** false **3c.** true **3d.** true **3e.** false

Page 32 **1.** 12 days **2.** 72 toy soldiers **3.** $163.83 **4.** no

Page 33 **5.** 5 ways **6.** 45 pairs **7.** 32 whole numbers **8.** 20 times **9.** 6 ways **10.** 24 combinations

UNIT 9

Page 34 **1.** 18 correct **2.** $1.35 **3.** $9.44

Page 35 **4.** 75 steps **5.** 50 sit-ups **6a.** 200 km **6b.** 150 km **6c.** 312.5 km **7.** 8 large pizzas **8.** 25 runs **9.** 35 people **10.** 15 tables **11.** 72 pictures

Page 36 **1.** 35% **2.** 60% **3.** 52%

Page 37 **4.** $9.00 **5.** $4,000.00 **6.** $120.00 **7.** 90 g **8.** $400.00 **9.** 9,120,000 kL **10.** $1,440.00 **11.** 3 incorrect **12.** $60

UNIT 10

Page 38 **1.** $70,772.40 **2.** 8 buses **3.** 18.3 mpg

Page 39 **4.** 131,400,000,000,000 **5.** 5.6 h **6.** 36 km **7.** 0.74 in. **8.** a box of oranges **9.** about 10 more **10.** $5000 **11.** 60,000 cm²

Page 40 **1.** Vancouver, 5 points **2.** 846 passengers

Page 41 **3.** Chip-Drive Co. **4.** yes **5.** 35 min **6.** 52% **7.** $642.00

UNIT 11

Page 42 **1.** 1 L of gas in the U.S.

Page 43 **2.** 1 min 3.46 s **3.** about 1028 times **4.** about 1 point **5.** about 2,964,761,900

Page 44 **1.** 91 squares

Page 45 **2.** 457 total bases **3.** 176 sq yd **4.** 1200 cm² **5.** 9 problems **6.** 45 numbers **7.** 4:15 P.M. **8.** 48 sq. mi

UNIT 12

Page 46 **1.** b **2.** b

Page 47 **3.** b **4.** b **5.** a **6.** a **7.** c **8.** a

Page 48 **1.** $10.50 **2.** $405.00 **3.** 196 students

Page 49 **4.** $262.50 **5.** 58,702 males **6.** 10 bags **7.** 109.5 L **8.** 25 ft **9.** $3360

UNIT 13

Page 50 **1.** about 989.5 hours (Yukon River is 1979 mi) **2.** 200 mi (Kansas City is 600 mi from Minneapolis and 400 mi from Chicago) **3.** 1650 cats would weigh as much as one average elephant (cat: 10 lb; elephant: 16,500 lb) **4.** blue whale, 100 tons; plankton

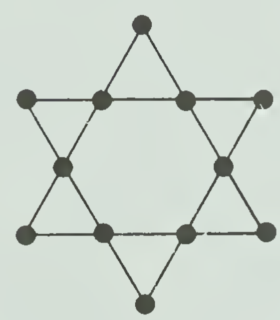
Page 51 **5.** 1 ft 2⅞ in. (1984 high jump: 7 ft 8½ in. by Dietmar Morenburg of W. Germany) (1932 high jump: 6 ft 5⅝ in. by Duncan McNaughton of Canada) **6.** Answers will vary. **7.** 1980 U.S. population: 226,504,825; 1970 U.S. population: 203,235,298; 1960 U.S. population: 179,323,175 **8.** Answers will vary. **9.** more than 28 lb heavier **10.** Arizona admitted in 2/14/12; Alaska admitted in 1/3/59 **11.** U.S. has about 12,383 mi of coastline; Canada, 17,860 mi **12.** about 18,000 mi

Page 52-53 Answers vary according to the source used.

UNIT 14

Page 54 **1.** 468 cm **2.** 36 problems **3.** 4 cylinder and 8 cylinder **4.** 6 h

Page 55 **5.** 10 cm **6.** 99 palindromes **7.**



8. Clearbrook 22, Mountainview 44, Shadylane 132
9. Melanie, 0.3 h or 18 min faster

Page 56 **10.** square, 4 cm² **11.** 8 cm, 4 cm **12.** 6 cars **13.** not enough information **14.** about 26 times

Page 57 **15.** 9 months: $1.33; 12 months: $1.33; 18 months: $1.28; 24 months: $1.17; no **16.** 35 **17.** 16 blocks **18.** 4 blocks

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